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RESEARCH ARTICLE

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Optimising two-stage robust supplier selection and order allocation problem under risk-averse criterion

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ABSTRACT
This paper studies the supplier selection and order allocation (SS&OA) problem, where risks include a series of disruption scenarios with uncertain probability of occurrence. It is a challenge for industry decision-makers to balance the average cost and the level of risk under the ambiguity set for probabilities. To address this challenge, a two-stage distributionally robust (DR) Mean-CVaR model is presented for the SS&OA problem. A procedure is developed for constructing the ambiguity set, and Polyhedral and Box ambiguity sets are constructed to characterise the uncertain probabilities. The worst-case Mean-CVaR criterion is employed for the second-stage cost within the ambiguity set to trade off the expected cost and CVaR value. Three measures are incorporated to increase the resilience of the supply chain. The proposed robust model is reformulated into two mixed-integer linear programming models. A real case of the Huawei cell phone manufacturer is used to illustrate the validity of the proposed approach in numerical settings. Experimental results show that the new optimising approach can provide a robust SS&OA solution to immunise against the influence caused by uncertain probabilities. By comparative analyses, some management insights are obtained for industry decision-makers.

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1. Introduction

The SS&OA problem plays an important role in global supply chain management and is becoming more profound (Saputro, Figueira, and Almada-Lobo 2021). With globalisation and the emergence of the extended enterprise of interdependent organisations, a new business strategy, i.e. outsourcing, has been widely accepted by large manufacturers (Dolgui and Proth 2013; Wu et al. 2013). For example, Boeing, the world's largest aircraft manufacturer, makes only the cockpit and wingtips and outsources other components to different enterprises. By outsourcing, manufacturers can reposition themselves, save costs, elevate their competitive capabilities, and reallocate various resources to focus on areas that best reflect their relative strengths (Harland et al. 2005; Moghaddam 2015). However, before manufacturers proceed with outsourcing, they must determine the best supplier portfolio and allocate the optimal order quantity among the selected suppliers, i.e. address the SS&OA problem. In this context, it is crucial to deal with the SS&OA problem effectively. Recently, the SS&OA problem has attracted the attention of many scholars, including Bodaghi, Jolai, and Rabbani (2018) and Mohammed, Harris, and Govindan (2019).

It is necessary to incorporate the disruption risk faced by the manufacturer into the SS&OA problem. The COVID-19 outbreak has affected global and local economies on a large scale. It has brought significant risks of disruption of the manufacturer's supply chain (Ivanov and Dolgui 2021a). In addition, among Fortune 1000 companies, 94% reported that COVID-19 resulted in supply chain disruptions (Fortune 2020). Disruption risks can potentially bring significant downsides to the supply chain (Dolgui and Ivanov 2021). In general, supply chain risks are classified into two categories: operational risks and disruption risks (Tang 2006). Operational risks refer to those inherent uncertainties of demand, cost, capacity, the absence of key personnel and power outages. The features of operational risks are that they are caused by usual events with medium to high probability of occurrence, with low impact, and with only short-term negative effects (Hosseini and Barker 2016). Compared with operational risks, disruption risks are usually caused by major disruptive events, such as natural disasters or

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human-made threats with a low likelihood of occurrence (Hosseini, Ivanov, and Dolgui 2019a). However, it is worth noting that once a disruption occurs, it may have long-term negative impacts on supply chain operations (Ivanov 2021). With the studied SS&OA problem, the disruption risks are taken into account and a series of disruption scenarios are used to describe them.

In the existing research, the probability of occurrence of disruption scenarios are assumed to be deterministic, such as Sawik (2013b), Torabi, Baghersad, and Mansouri (2015), Ni, Howell, and Sharkey (2018). In this case, the classical stochastic optimisation (SO) method can be used to model the SS&OA problem. However, regarding the probability estimation of occurrences for disruption scenarios, there are the following two difficulties: (i) sufficient historical data related to rare events are difficult to obtain, and (ii) the available historical data usually cannot fully reflect reality (Aldrighetti et al. 2021). Therefore, it is usually unrealistic to assume that the probabilities of occurrence for disruption scenarios are deterministic. How should the decision-maker optimise the SS&OA problem if they face uncertain probabilities of occurrence for disruption scenarios? This is a critical and interesting issue to be resolved. The recently developed distributionally robust optimisation (DRO) method, an attractive optimisation method to deal with uncertain probability, provides a useful optimisation framework in which decision-makers do not require precise distributions and only use partial distribution information (Zhang et al. 2022). In the DRO method, a so-called ambiguity set is employed to characterise the uncertain probabilities of occurrence for disruption scenarios. The optimisation is based on the worst-case distribution within the ambiguity set (Delage and Ye 2010). In this paper, we use the DRO method to address the uncertain probabilities of occurrence for disruption scenarios and further cope with disruption risks in the SS&OA problem.

Under the uncertain probabilities of occurrences for disruption scenarios, the worst-case Mean-CVaR criterion is more suitable for risk-averse SS&OA decision-makers. There are three common decision criteria, i.e. expected (Mean), value-at-risk (VaR), and conditional value-at-risk (CVaR), for decision-making in the production research literature. Among them, CVaR, which is defined based on VaR, satisfies the following axioms: convexity, monotonicity, translation invariance, and positive homogeneity, and usually performs better than VaR (Rockafellar and Uryasev 2000). However, the risk-averse decision-maker may need to balance the average cost with the level of risk. To address this issue, based on the uncertain probabilities of occurrence for disruption scenarios, this paper develops the worst-case Mean-CVaR criterion to seek the trade-off between the expected cost and CVaR value about the post-disruption cost.

Currently, the supply chain is facing an increasing number of disruptive events (e.g. the COVID-19 outbreak, climate extremes). To make the supply chain more efficient and effective in this changing environment, some advanced ideas have been proposed as a guide in the recent literature. For example, viability is the ability of the supply chain to maintain itself and survive in a changing environment through the redesigning of structures and replanning of performance with long-term impacts (Ivanov and Dolgui 2020). A viable supply chain is a dynamically adaptable and structurally changeable, a value-added network that is able to (i) react agilely to positive changes, (ii) be resilient in absorbing negative events and recover after disruptions, and (iii) survive in times of long-term global disruptions (Ivanov 2020). A reconfigurable supply chain (the X-network) expresses a network designed in a cost-efficient, responsive, sustainable, and resilient manner that is increasingly data-driven and dynamically adaptable and capable for rapid structural changes in both physical and cyber space (Dolgui, Ivanov, and Sokolov 2020b). A reconfigurable supply chain exhibits four distinctive features, resilience, leagility, sustainability, and digitalisation, which mutually enhance each other. Resilience plays an essential role in both viability and reconfigurability (Ivanov 2021; Ivanov and Dolgui 2021b).

It is necessary to incorporate resilience measures into the SS&OA problem. The vulnerability of supply chain networks has increased due to globalisation of trade (Dixit, Seshadrinath, and Tiwari 2016) and unexpected natural disasters (Torabi, Baghersad, and Mansouri 2015). In this background, resilience in the supply chain is widely concerned against unexpected disruption risks. The following two famous examples also verify the significance of resilience. Philips, a semiconductor supplier located in Mexico, supplies semiconductors to manufacturers, including Nokia and Ericsson. Philip's supply was disrupted due to a sudden large fire, and because of the disruption, Ericsson lost $400 million, while Nokia suffered less because of cooperation with a backup supplier (Latour 2001). After a magnitude nine earthquake and tsunami that struck Japan in 2011, Toyota and Nissan tried to cooperate with suppliers who were geographically dispersed rather than those who were in a shorter distance zone (Hosseini et al. 2019b). Inspired by these scenarios, this paper adds three resilience measures that increase the resilience of the supply chain in the SS&OA problem.

Motivated by the above research, this paper addresses the following questions related to the resilient SS&OA problem: (i) How can the uncertain probabilities of occurrence be addressed for disruption scenarios? (ii) How can the average cost and the level of risk be balanced? (iii) Do the uncertain probabilities have a larger effect on the optimal SS&OA solution? To resolve the above questions, a two-stage DR Mean-CVaR model is proposed for the resilient SS&OA problem under disruption risks to minimise the trade-off between the expected cost and CVaR value. To the best of our knowledge, this work is the first to address the uncertain probabilities of occurrence for disruption scenarios in the SS&OA problem. The contributions of this paper to the production system can be summarised as follows:

- First, this paper uses a series of disruption scenarios to describe disruption risks and finds the optimal SS&OA scheme under uncertain probabilities of occurrence for disruption scenarios. A new procedure for constructing an ambiguity set and polyhedral and box ambiguity sets are developed for characterising the uncertain probabilities. To the best of our knowledge, this paper is the first to study the SS&OA problem based on the uncertain probabilities of occurrence for disruption scenarios.
- Second, this paper develops a novel decision aid model for the SS&OA problem in production systems. The worst-case Mean-CVaR criterion of the second-stage cost over the ambiguity set is utilised to help the manufacturer trade off the expected cost and CVaR value. More importantly, by employing the Lagrange and linear duality theories, the developed model is reformulated into MILP forms for the convenience of a large audience in production research.
- Third, this paper discusses the real-life application of the proposed optimisation approach in production systems. The manufacturer of Huawei cell phones in Changsha acts as a case study to demonstrate the validity of the proposed optimisation approach. Experimental results show that the proposed optimisation approach is feasible and effective for the SS&OA problem under uncertain probabilities of occurrence for disruption scenarios.

The rest of this paper is organised as follows. Section 2 briefly reviews the related literature. Section 3 gives the problem statement in detail and develops a new two-stage DR Mean-CVaR model for the resilient SS&OA problem. Section 4 analyses and reformulates the proposed model, constructs two different ambiguity sets, and reformulates the computationally tractable robust counterpart of the original model. In Section 5, the case study used to demonstrate the validity of the proposed optimisation approach is explored. In Section 6, a few management insights for decision-makers in the industry are reviewed, and the conclusion is given in Section 7.

2. Literature review

This section presents the literature review, shows the existing research gaps, and highlights the contributions. The literature review focuses on the uncertainty and decision criteria in the SS&OA problem.

Literature that incorporates uncertainty into the SS&OA problem includes Nazari-Shirkouhi et al. (2013), who developed an interactive two-phase fuzzy multi-objective linear programming method for solving the SS&OA problem. They considered the fuzzy degree of satisfaction of each objective function for the decision-maker and specified a piecewise linear membership function for each objective function. When Moheb-Alizadeh and Handfield (2018) studied the sustainable SS&OA problem, they considered the stochastic log-normal demands and developed a stochastic sustainable SS&OA model with chance constraints. They converted the stochastic constraints into their deterministic equivalents for the predetermined confidence levels via the inverse of the cumulative distribution function. Babbar and Amin (2018) proposed a multi-objective SS&OA model, where uncertainty appears in two areas. One area concerns the unit cost and demand, which are considered uncertain parameters based on finite stochastic scenarios. Another area concerns the importance levels among supplier evaluation criteria, which are modelled as trapezoidal fuzzy numbers in applying the QFD method to select suppliers. Mirzaee, Naderi, and Pasandideh (2018) studied a bi-objective generalised SS&OA problem, regarded the satisfaction level of each objective as a fuzzy parameter, assigned a linear membership function for each objective, and developed an effective pre-emptive fuzzy goal programming approach to solve this bi-objective model. Bodaghi, Jolai, and Rabbani (2018) developed a multi-objective model for the SS&OA problem, assigned a fuzzy linear affiliation function to each objective, and used the fuzzy analytic network process (FANP) method to transform the multi-objective model into a single-objective model. In addition, in the process of estimating model parameters, they assumed demand obeys stochastic Gaussian distribution (given the expectation and standard deviation) and used the expected demand quantity reduction penalty (EDQRP) method to estimate the contracted ordering intervals and ordering capacity intervals of suppliers. Mohammed, Harris, and Govindan (2019) established a multi-objective optimisation model for a sustainable SS&OA problem, considered the importance levels among supplier evaluation criteria (conventional, green, and social) as triangular fuzzy numbers, and proposed an integrated fuzzy AHP-fuzzy TOPSIS to assess and rank suppliers. Jia, Liu, and Bai (2020) studied the sustainable SS&OA problem based on uncertain unit cost, CO_2 emissions, demand, and supply capacity, where only partial distribution information is known. They developed a distributionally robust sustainable SS&OA goal programming model. Firouzi and Jadidi (2021) considered uncertain parameters, such as demand, fixed ordering cost, defective and late delivery rates, and supply capacity. These uncertain parameters are identified with triangular or trapezoidal fuzzy numbers. Based on fuzzy set theory, they developed a fuzzy multi-objective model for the SS&OA problem. A multi-objective multistage programming model was proposed for sustainable SS&OA problems, and fuzzy set theory was utilised to capture the relative importance of decision-makers who take part in the decision-making team (Wu, Gao, and Barnes 2022). There are still numerous researchers who have studied uncertainty in the SS&OA problem, such as Kannan et al. (2013), Suprasongsin, Yenradee, and Huynh (2020), and Nasr et al. (2021), but this literature is not going to be reviewed here. The above literature that addresses parameter uncertainty in the SS&OA problem is based on the following assumption, i.e. the true distribution of uncertain parameters must be known or estimated exactly and does not fully capture the probability uncertainty of occurrence for disruption scenarios. However, it is usually difficult for decision-makers in industry to estimate the true distribution. This is the main limitation of the above literature in handling the parameter uncertainty of the SS&OA problem.

In the literature, multiple different criteria are set as objective functions to make the optimal SS&OA decision. Jia, Liu, and Bai (2020) set four objectives, one of which is to minimise the total cost, and used goal programming to solve the multi-objective SS&OA problem. Additionally, there are a large number of scholars who employ the expected cost criterion to make SS&OA decisions. Based on multiple disruption scenarios, Hosseini et al. (2019b) developed a bi-objective mixed integer linear programming (MILP) model, where one objective is to minimise the total expected cost. Torabi, Baghersad, and Mansouri (2015) and Khalili, Jolai, and Torabi (2017) considered some critical parameters (such as demands and costs) as fuzzy numbers in response to operational risks and set the expected cost and CVaR based on finite disruption scenarios as the objective functions to make decisions. When Vahidi, Torabi, and Ramezankhani (2018) studied the sustainable SS&OA problem, they established a bi-objective two-stage mixed possibilistic-stochastic programming model under operational and disruption risks. In their model, the first objective is to minimise the total sustainability and resilience scores of the selected suppliers, the demand is treated as a fuzzy parameter to mitigate operational risks, and the second objective is to use scenario-based expected costs to control disruption risks. Jabbarzadeh, Fahimnia, and Sabouhi (2018) also used the expected cost criterion to make decisions, investigated resilient and sustainable supply chain design under disruption scenarios, and developed a bi-objective optimisation model. Their first objective minimises the expected total costs of the supply chain, and the second objective maximises the aggregate weighted sustainability scores of all selected suppliers under different scenarios. When Sanci et al. (2021) investigated how to choose the best mitigation strategy against supply disruption risk, they randomly generated some disruption scenarios by using a scenario tree, estimated the probability of each scenario occurring, and set an objective function that minimises the expected cost. Alternatively, some scholars have employed the mean-risk criterion for making decisions. For instance, Sawik (2013a) built two SS&OA models based on disruption risks to minimise the expected cost for risk-neutral performance and the CVaR performance for risk-averse decision-makers. Under conditions of risk-neutral and risk-averse, Sawik (2013b) built three different SS&OA models to minimise the expected cost, CVaR, and Mean-CvaR, where the third model can be used to balance the expected cost and risk tolerance. In the reviewed literature, except Sawik (2013b), most studies used only one criterion (Mean, VaR, or CVaR) as an objective function to make the optimal SS&OA decision. Although Sawik (2013b) used Mean-CVaR, the proposed model was based on the deterministic probabilities of occurrences for disruption scenarios. These approaches do not balance the average cost and the level of risk under uncertain probabilities for the risk-averse decision-maker. This is the second limitation of the reviewed literature.

To identify the research gaps of the existing studies and to clarify the innovations of this paper, the related literature is classified in Table 1, which summarises the following three research gaps: (i) there is no literature that investigates the SS&OA problem under uncertain probabilities of occurrence for disruption scenarios hitherto; (ii) no literature applies the worst-case Mean-CVaR criterion to the SS&OA problem for the risk-averse decision-maker until now; (iii) there is only very sparse literature on optimising the SS&OA problem with the DRO method. To address these gaps, this paper considers the uncertain probabilities of occurrence for disruption scenarios and presents a two-stage DR Mean-CVaR model under disruption risks. In summary, this study departs significantly from previous studies and is a step forward in solving the SS&OA problem.

Table 1. A review of relevant works in the literature.

		Uncertainty		Resilience			Optimisation method			
Reference	Disruption risks	PDSO	Other	BS	GS	Other	SO	FO	DRO	Decision criterion
Kannan et al. (2013)			√					√		Total cost
Nazari-Shirkouhi et al. (2013)			√					√		Total cost
Sawik (2013a)	√		√				√			Expected cost, CVaR
Sawik (2013b)	√		√	√		√	√			Expected cost, CVaR
Torabi, Baghersad, and Mansouri (2015)	√		√	√			√	√		Expected cost
Khalili, Jolai, and Torabi (2017)	√		√	√		√	√	√		CVaR
Moheb-Alizadeh and Handfield (2018)			√				√			Total cost
Mirzaee, Naderi, and Pasandideh (2018)			√					√		Total cost
Babbar and Amin (2018)			√					√		Total cost
Ni, Howell, and Sharkey (2018)	√			√		√	√			Expected cost
Jabbarzadeh, Fahimnia, and Sabouhi (2018)	√		√	√		√	√	√		Expected cost
Vahidi, Torabi, and Ramezankhani (2018)	√		√	√			√	√		Expected cost
Bodaghi, Jolai, and Rabbani (2018)			√				√	√		Total cost
Mohammed, Harris, and Govindan (2019)			√					√		Total cost
Hosseini et al. (2019b)	√			√	√	√	√			Expected cost
Suprasongsin, Yenradee, and Huynh (2020)			√					√		Total cost
Jia, Liu, and Bai (2020)			√						√	Total cost
Nasr et al. (2021)			√					√		Total cost
Firouzi and Jadidi (2021)			√					√		Total cost
Sanci et al. (2021)	√			√		√	√			Expected cost
Wu, Gao, and Barnes (2022)			√					√		Expected cost
This research	√	√		√	√	√			√	Worst-Case Mean-CVaR

Note: BS, GS, PDSO, and FO denote backup supplier, geographical segregation, probability of occurrence for disruption scenario, and fuzzy optimisation, respectively.

3. Model development

In this section, the SS&OA problem is described, the objective and resilience constraints are constructed, and a new two-stage DR mean-CVaR model is developed.

3.1. Problem statement

The manufacturer usually outsources its production to a set of preidentified suppliers, including the two groups of main and backup suppliers. These suppliers are often exposed to many disruptive events, such as floods, earthquakes, and hurricanes. Once one supplier regularly faces disruptive events, this supplier may continue or fail to operate (disrupted). Therefore, the manufacturer faces a range of disruption scenarios, and each scenario contains two groups of suppliers: continuously operational suppliers and disrupted suppliers. In addition, the probability of occurrence for each disruption scenario is usually uncertain. The manufacturer needs to take various measures to increase supply chain resilience and to hedge against disruptions. Moosavi and Hosseini (2021) pointed out that cooperating with a backup supplier is a valuable resilience and response strategy during supply chain disruption. Inspired by this, the first resilience measure is to cooperate with both main and backup suppliers. Aldrighetti et al. (2021) mentioned that it is important to ask for more products than agreed upon from regular contracted suppliers (i.e. surplus supply from non-disrupted main suppliers). Therefore, the second resilience measure is that the non-disrupted main suppliers can provide the surplus supply to the manufacturer. Motivated by Hosseini et al. (2019b), the third measure is to set the shortest segregation distance between any two selected suppliers and the total distance of all selected suppliers. Figure 1 facilitates the reader's understanding of the resilient SS&OA problem.

In Figure 1, the example of a manufacturer cooperating with six suppliers is used to demonstrate the network structure of the SS&OA problem. From Figure 1, the manufacturer cooperates with two main suppliers and two backup suppliers in the pre-disruption stage. The actual distances between any two selected suppliers are not less than the required, least the segregation distance. Only these two main suppliers produce and distribute the products before the disruption. In the post-disruption stage, main supplier-2 reduces the production and distribution quantities of products. For main supplier-1, in addition to completing specified quantities, it also produces surplus product for supply. Backup supplier-1 produces and distributes some products. Backup supplier-2 still does not produce the products.

The SS&OA problem with a series of disruption scenarios is a classical two-stage problem. These two stages refer to pre-disruption and post-disruption, respectively. The decision-maker determines the decisions of

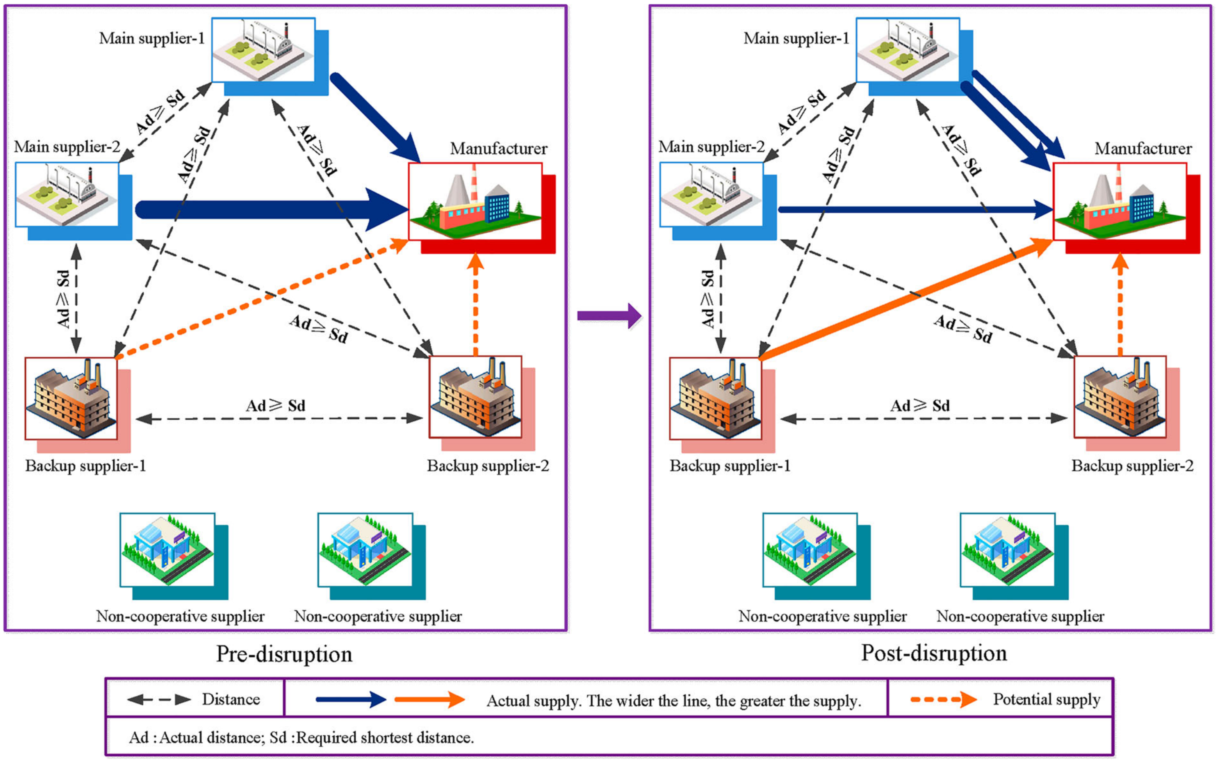


Figure 1. Structure of the SS&OA problem studied.

pre-disruption before knowing the realised scenario while determines the decisions of post-disruption after knowing the realised scenario to compensate for the decisions made at pre-disruption. Therefore, we need to establish a two-stage optimisation model. The decision-maker needs to make the following two-stage decisions to minimise the objective. In the first stage, i.e. pre-disruption, it is required to identify (i) which suppliers are selected as main suppliers; (ii) which suppliers are selected as backup suppliers; and (iii) the order quantity at each main supplier. In the second stage, i.e. post-disruption, it is required to determine (i) the product quantity received from the disrupted main suppliers; (ii) the product quantity received from the non-disrupted backup suppliers; and (iii) the surplus product quantity received from the non-disrupted main suppliers. The surplus product quantity is the additional product quantity from the non-disrupted main suppliers in addition to the order quantity specified in the first stage.

The notations and their definitions used in our optimisation model are as follows:

Sets and indices:

I: Set of suppliers, indexes $i, j \in I$;
S: Set of disruption scenarios, index $s \in S$;
I_s: Set of suppliers that are non-disrupted under scenario s;
$\bar{I}_s$: Set of suppliers that are disrupted under scenario s;
E_i: Set of possible disruptive events that supplier $i (i \in I)$ may face. For example, for supplier i, E_i might be Tsunamis (e_{i1}), Floods (e_{i2}), Earthquakes (e_{i3}), Hurricane (e_{i4});
e_{in}: A certain disruptive event at supplier i, $e_{in} \in E_i$, $n \in [|E_i|]$, where $|E_i|$ is the cardinality of the set E_i.

Parameters:

d_{ij}: The distance between suppliers i and j;
Sd: Least segregation distance between every pair of suppliers;
TD: Least total segregation distance among all selected suppliers;
D: Manufacturer's demand;
C_i: Fixed cost of contracting with supplier i as a main supplier;
C'_i: Fixed cost of contracting with supplier i as a backup supplier ($C'_i \geq C_i$);
L_i: Purchasing and transportation (P&T) cost per product from main supplier i;
L'_i: P&T cost per product from backup supplier i;
L''_i: P&T cost per surpass product from non-disrupted main supplier $i (L''_i \geq L'_i \geq L_i)$;

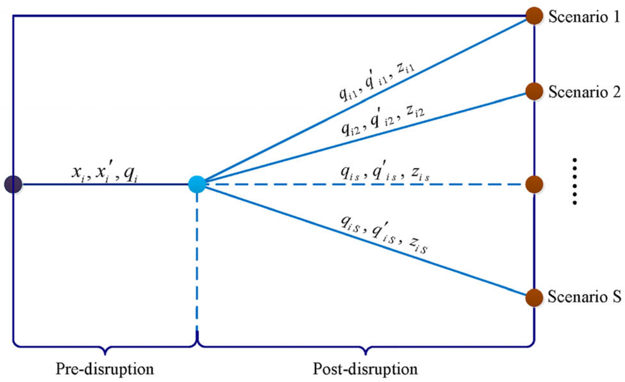


Figure 2. Decision variables in the first and second stages.

Ca_i: Production capacity of supplier i under normal (i.e. working) conditions;

θ_{is}: The percentage of normal capacity of supplier $i(i \in \bar{I}_s)$ under disruption scenario s;

$\pi_{e_{in}}$: Occurrence likelihood of disruptive event $e_{in}(e_{in} \in E_i, n \in [|E_i|])$ that supplier i may face;

π_i: Disruption probability of supplier i;

P_s: The probability of occurrence for disruption scenario s;

R: Maximum number of main suppliers allowed to be contracted in the normal situation;

M: A very large constant.

First stage (pre-disruption) decision variables:

x_i: Binary variables, 1 if supplier $i(i \in I)$ is selected as the main supplier, and 0, otherwise;

x'_i: Binary variables, 1 if supplier $i(i \in I)$ is selected as backup supplier, and 0, otherwise;

q_i: Order quantity from the main supplier $i(i \in I)$ at the stage of pre-disruption.

Second stage (post-disruption) decision variables:

q_{is}: Quantity that the manufacturer will receive from main supplier $i(i \in I)$ at the stage of post-disruption under scenario s;

q'_{is}: Quantity that the manufacturer will receive from backup supplier $i(i \in I)$ at the stage of post-disruption under scenario s;

z_{is}: Surplus quantity that the manufacturer will receive from the non-disrupted main supplier $i(i \in I)$ at the post-disruption under scenario s.

The relationships between decision variables in these two stages are shown in Figure 2.

3.2. Objective function

The objective includes two parts. The first part, which corresponds to the first stage, is built as

$$\mathrm{TC}_1(\boldsymbol{x}, \boldsymbol{x}', \boldsymbol{q}^1) = \sum_{i \in I} C_i x_i + \sum_{i \in I} L_i q_i + \sum_{i \in I} C'_i x'_i,$$

where $\boldsymbol{x} = (x_i)_{i \in I}$, $\boldsymbol{x}' = (x'_i)_{i \in I}$, $\boldsymbol{q}^1 = (q_i)_{i \in I}$. The first item of $\mathrm{TC}_1(\boldsymbol{x}, \boldsymbol{x}', \boldsymbol{q}^1)$ is the cost of contracting with the main suppliers. The second item is the cost of contracting with the backup suppliers. The third item is the P&T cost.

The second part is the worst-case Mean-CVaR value of the second-stage cost. In the real world, which is full of uncertainty, obtaining an exact probability distribution is a very difficult task. It is logical to assume that the probabilities of occurrence for disruption scenarios are uncertain and belong to an ambiguity set $\mathcal{P}$. Here, setting $\mathbf{P} = (\mathrm{P}_s)_{s \in S}$, the discrete probability distribution $\mathbf{P} \in \mathcal{P}$ is obtained. In this context, to help the decision-maker balance the average cost and the level of risk, the following worst-case Mean-CVaR criterion is developed:

$$\max_{\mathbf{P} \in \mathcal{P}} \{\alpha \mathbf{E}_{\mathbf{P}}[\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))] + (1-\alpha)\,\mathbf{CVaR}_{\varepsilon,\mathbf{P}}[\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))]\},$$

where $\boldsymbol{q}^2 = (q_{is})_{i \in I, s \in S}$, $\boldsymbol{q}^3 = (q'_{is})_{i \in I, s \in S}$, $\boldsymbol{z} = (z_{is})_{i \in I, s \in S}$, α is the trade-off parameter, ε is the confidence level parameter, and $\mathrm{TC}_2\,(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))_{s \in S}$ is the value function of the second-stage problem.

As a consequence, the objective is formulated as:

$$\mathrm{TC}_1(\boldsymbol{x}, \boldsymbol{x}', \boldsymbol{q}^1) + \max_{\mathbf{P} \in \mathcal{P}} \{\alpha \mathbf{E}_{\mathbf{P}}[\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))] + (1-\alpha)\,\mathbf{CVaR}_{\varepsilon,\mathbf{P}}[\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))]\}.$$

3.3. Resilience constraints

To increase the resilience of the supply chain, two types of resilience constraints are constructed. First, the sum of the distances among all selected suppliers for cooperation is not less than the shortest total segregation distance TD. The sum of the distances between the main suppliers can be measured by $\sum_{i \in I} \sum_{j \in I, i<j} x_i x_j d_{ij}$. Similarly, the sum of the distances between the main suppliers and backup suppliers and the sum of the distances between the backup suppliers can be represented by $\sum_{i \in I} \sum_{j \in I} x_i x'_j d_{ij}$

and $\sum_{i\in I}\sum_{j\in I,i<j} x'_i x'_j d_{ij}$, respectively. Using the above formulas, the following constraint is built:

$$TD \le \sum_{i\in I}\sum_{j\in I,i<j} x_i x_j d_{ij} + \sum_{i\in I}\sum_{j\in I} x_i x'_j d_{ij} + \sum_{i\in I}\sum_{j\in I,i<j} x'_i x'_j d_{ij}. \quad (1)$$

Second, the distance between any two selected suppliers is greater than or equal to the least segregation distance Sd, which is represented as the following constraints:

$$Sd \le d_{ij} + M(2 - x_i - x_j), \quad \forall i,j \in I, i \ne j, \quad (2)$$

$$Sd \le d_{ij} + M(2 - x_i - x'_j), \quad \forall i,j \in I, i \ne j, \quad (3)$$

$$Sd \le d_{ij} + M(2 - x'_i - x'_j), \quad \forall i,j \in I, i \ne j. \quad (4)$$

For convenience, TD and Sd are referred to collectively as the resilience distances.

3.4. Two-stage DR Mean-CVaR model for resilient SS&OA problem

Based on the analysis in the above subsections, the two-stage DR Mean-CVaR model is formally developed as follows:

$$\begin{aligned}\min \quad & \mathrm{TC}_1(\boldsymbol{x}, \boldsymbol{x}', \boldsymbol{q}^1) \\ & + \max_{\mathbf{P}\in\mathcal{P}} \{\alpha \mathbf{E}_{\mathbf{P}}[\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))] \\ & + (1-\alpha)\,\mathbf{CVaR}_{\varepsilon,\mathbf{P}}[\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))]\}\end{aligned}$$

$$\text{s.t.} \quad \sum_{i\in I} q_i = D, \quad (5)$$

$$\sum_{i\in I} x_i \le R, \quad (6)$$

$$x_i + x'_i \le 1, \forall i \in I, \quad (7)$$

$$0 \le q_i \le x_i Ca_i, \quad x_i, x'_i \in \{0,1\}, \quad \forall i \in I, \quad (8)$$

$$\text{Constraints}(1) - (4).$$

Constraint (5) guarantees satisfying the manufacturer's demand at the first stage. Constraint (6) indicates that the number of selected main suppliers for cooperation in the pre-disruption stage should not be more than R. Constraint (7) means that no supplier can be selected as both the main supplier and backup supplier at the same time. Constraint (8) limits the range of decision variables at the first stage. The order quantity q_i from the main supplier i should be greater than or equal to 0 and less than or equal to production capacity Ca_i. Given i, both x_i and x'_i are Boolean. $\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))$ in the objective is the optimal value of the second stage programming model:

$$\min \sum_{i\in I_s} L'_i q'_{is} - \sum_{i\in \bar{I}_s} L_i(q_i - q_{is}) + \sum_{i\in I_s} z_{is} L''_i$$

$$\text{s.t.} \sum_{i\in I_s}(q_i + q'_{is}) + \sum_{i\in \bar{I}_s} q_{is} + \sum_{i\in I_s} z_{is} = D, \forall s \in S, \quad (9)$$

$$q_{is} + z_{is} \le Ca_i x_i, \forall i \in I_s, s \in S, \quad (10)$$

$$q'_{is} \le Ca_i x'_i, \forall i \in I_s, s \in S, \quad (11)$$

$$q_{is} \le \theta_{is} Ca_i x_i, \forall i \in \bar{I}_s, s \in S, \quad (12)$$

$$q_{is} \le q_i, \forall i \in \bar{I}_s, s \in S, \quad (13)$$

$$q_i = q_{is}, \forall i \in I_s, s \in S, \quad (14)$$

$$q'_{is} = 0, \forall i \in \bar{I}_s, s \in S, \quad (15)$$

$$q_{is}, q'_{is}, z_{is} \ge 0, \forall i \in I, s \in S. \quad (16)$$

The objective is to minimise the sum of P&T costs $\sum_{i\in I_s} L'_i q'_{is}$, $\sum_{i\in \bar{I}_s} L_i(q_{is} - q_i)$, and $\sum_{i\in I_s} z_{is} L''_i$ after the disruption scenario occurs. Constraint (9) means that the manufacturer's demand should be satisfied under each scenario. Constraint (10) implies that the delivery quantity from each non-disrupted main supplier is not more than the supplier's production capacity. Constraint (11) implies that the delivery quantity from each non-disrupted backup supplier is not more than the supplier's production capacity. Constraint (12) denotes that the delivery quantity from each selected disrupted main supplier should not be more than the supplier's production capacity. Constraint (13) indicates that the quantity of products delivered by the disrupted main supplier at the post-disruption stage should be less than or equal to the quantity of products contracted at the pre-disruption stage. Constraint (14) means that the quantity of products delivered by the non-disrupted main supplier at the post-disruption stage should be equal to the quantity of products contracted at the pre-disruption stage. Constraint (15) means that the disrupted suppliers cannot be used as backup suppliers under each scenario. Constraint (16) denotes the types of decision variables at the second stage.

Remark 3.1: Following the idea of Vahidi, Torabi, and Ramezankhani (2018), we set equality constraint (9). The established model avoids the case where the disruption event destroys so much capacity that the suppliers cannot fulfil the demand D by choosing the number of suppliers to cooperate with. This is mainly because as long as there are enough candidate suppliers, a sufficient number of suppliers can be selected for cooperation to meet demand D.

According to Aldrighetti et al. (2021), who systematically analysed the costs involved in the supply chain network design with disruption risks, the costs of the model could be divided into two categories: cooperation costs (i.e. $\sum_{i\in I} C_i x_i$ and $\sum_{i\in I} C'_i x'_i$) and P&T costs (i.e. $\sum_{i\in I} L_i q_i$, $\sum_{i\in I_s} L'_i q'_{is}$, $\sum_{i\in \bar{I}_s} L_i(q_i - q_{is})$ and $\sum_{i\in I_s} z_{is} L''_i$). Among them, $\sum_{i\in I} C'_i x'_i$, $\sum_{i\in I_s} L'_i q'_{is}$, and $\sum_{i\in I_s} z_{is} L''_i$ belong to resilience costs.

4. Model analysis

In this section, the objective function is reformulated, the procedure for constructing ambiguity sets is developed, robust counterparts are derived, and the computationally tractable form of the two-stage DR Mean-CVaR model is presented.

4.1. Reformulating objective function

To find the computationally tractable form of the proposed two-stage DR model, next the second part of the objective function is reformulated.

First, in light of the definition of $\mathbf{CVaR}_{\varepsilon,\mathbf{P}}$ (Rockafellar and Uryasev 2000), the CVaR value of the second-stage cost, $\mathbf{CVaR}_{\varepsilon,\mathbf{P}}[\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))]$, can be represented as

$$\min_{\phi\in\mathbb{R}} \left\{ \phi + \frac{1}{1-\varepsilon} \mathbf{E}_{\mathbf{P}}[\max\{\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s)) - \phi, 0\}] \right\},$$

where $\varepsilon \in (0,1)$ is a confidence level parameter and reflects the probability that $\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))$ is lower than $\mathbf{VaR}_{\varepsilon,\mathbf{P}}$. Meanwhile, ε also measures the risk preferences of the decision-maker. The larger ε is, the more risk-averse the decision-maker is.

According to the above definition, there exist the following equivalent transformations:

$$\begin{aligned}
&\max_{\mathbf{P}\in\mathcal{P}} \{\alpha \mathbf{E}_{\mathbf{P}}[\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))] \\
&\qquad + (1-\alpha)\, \mathbf{CVaR}_{\varepsilon,\mathbf{P}}[\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))]\} \\
&= \max_{\mathbf{P}\in\mathcal{P}} \Big\{ \alpha \mathbf{E}_{\mathbf{P}}[\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))] \\
&\qquad + (1-\alpha) \min_{\phi\in\mathbb{R}} \Big\{ \phi + \frac{1}{1-\varepsilon} \mathbf{E}_{\mathbf{P}}[\max\{\mathrm{TC}_2(\boldsymbol{q}^2(s), \\
&\qquad \boldsymbol{q}^3(s), \boldsymbol{z}(s)) - \phi, 0\}] \Big\} \Big\}.
\end{aligned}$$

Second, according to Corollary 37.3.2 from Rockafellar (1970), the order of $\max_{\mathbf{P}\in\mathcal{P}}$ and $\min_{\phi\in\mathbb{R}}$ can be changed. Therefore, the above formula is reformulated equivalently as:

$$\begin{aligned}
&\min_{\phi\in\mathbb{R}} \max_{\mathbf{P}\in\mathcal{P}} \Big\{ \alpha \mathbf{E}_{\mathbf{P}}[\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))] \\
&\qquad + (1-\alpha) \Big\{ \phi + \frac{1}{1-\varepsilon} \mathbf{E}_{\mathbf{P}}[\max\{\mathrm{TC}_2(\boldsymbol{q}^2(s), \\
&\qquad \boldsymbol{q}^3(s), \boldsymbol{z}(s)) - \phi, 0\}] \Big\} \Big\}.
\end{aligned}$$

Given a disruption scenario $s \in S$, we denote $\mathrm{TC}_2(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))$ as $\mathrm{TC}_{2,s}(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s))$ and introduce auxiliary variables t_s to represent $\max\{\mathrm{TC}_{2,s}(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s)) - \phi, 0\}$. Because the probability of occurrence for disruption scenario s is P_s, the second part of the objective function is reformulated as the following form:

$$\min_{\boldsymbol{q}^2,\boldsymbol{q}^3,\boldsymbol{z},\phi} \max_{\mathbf{P}\in\mathcal{P}} \left\{ \alpha \sum_{s\in S} \mathrm{P}_s \mathrm{TC}_{2,s}(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s)) + (1-\alpha)\left\{ \phi + \frac{1}{1-\varepsilon} \sum_{s\in S} \mathrm{P}_s \mathrm{t}_s \right\} \right\}$$

$$\text{s.t.} \quad \sum_{i\in I_s} L'_i q'_{is} - \sum_{i\in \bar{I}_s} L_i(q_i - q_{is}) + \sum_{i\in I_s} z_{is} L''_i - \phi \le \mathrm{t}_s, \quad \forall s \in S, \tag{17}$$

$$0 \le \mathrm{t}_s, \quad \forall s \in S. \tag{18}$$

After reformulating the objective function, the proposed original model is equivalently represented as the following model:

$$\left\{ \begin{array}{ll}
\min\limits_{\varphi} & \mathrm{TC}_1(\boldsymbol{x}, \boldsymbol{x}', \boldsymbol{q}^1) \\
& + \max\limits_{\mathbf{P}\in\mathcal{P}} \Big\{ \alpha \sum_{s\in S} \mathrm{P}_s \mathrm{TC}_{2,s}(\boldsymbol{q}^2(s), \boldsymbol{q}^3(s), \boldsymbol{z}(s)) \\
& + (1-\alpha) \Big\{ \phi + \frac{1}{1-\varepsilon} \sum_{s\in S} \mathrm{P}_s \mathrm{t}_s \Big\} \Big\} \\
\text{s.t.} & \text{Constraints}(1) - (18),
\end{array} \right. \tag{19}$$

where $\varphi = (\boldsymbol{x}, \boldsymbol{x}', \boldsymbol{q}^1, \boldsymbol{q}^2, \boldsymbol{q}^3, \boldsymbol{z}, \boldsymbol{r}, \boldsymbol{t}, \phi)$.

4.2. Constructing ambiguity sets based on disruption scenarios

The probability distribution of disruption scenario occurrence is discrete and uncertain. In this section, in accordance with Qiu and Shang (2014), limited historical data and expert knowledge are utilised to construct Polyhedral and Box ambiguity sets to characterise the uncertain discrete probability distribution.

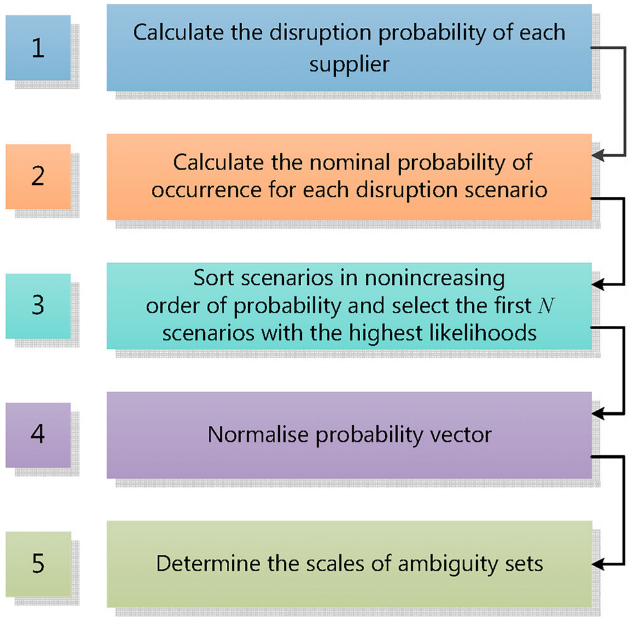


Figure 3. The procedure for constructing the Polyhedral and Box ambiguity sets.

The form of the polyhedral ambiguity set is:

$$\mathcal{P}_{\text{Polyhedral}} = \{\mathbf{P}|\mathbf{P} = \mathbf{P}^0 + \mathbf{A}\boldsymbol{\xi}, \boldsymbol{e}^{\mathrm{T}}\mathbf{A}\boldsymbol{\xi} \geq 0, ||\boldsymbol{\xi}||_1 \leq 1\}, \tag{20}$$

where $\mathbf{P}^0$ denotes the nominal distribution, which is the most likely distribution of the disruption scenario occurrence, $\mathbf{A} \in \mathbb{R}^{|S|\times|S|}$ is the scaling matrix of the polyhedral, $\boldsymbol{\xi}$ denotes the uncertain parameter vector, and $\boldsymbol{e}$ denotes the unit column vector.

The form of the Box ambiguity set is:

$$\mathcal{P}_{\text{Box}} = \{\mathbf{P}|\mathbf{P} = \mathbf{P}^0 + \boldsymbol{\xi}, \boldsymbol{e}^{\mathrm{T}}\boldsymbol{\xi} = 0, \boldsymbol{\xi}_L \leq \boldsymbol{\xi} \leq \boldsymbol{\xi}_U\}, \tag{21}$$

where $[\boldsymbol{\xi}_L, \boldsymbol{\xi}_U]$ denotes the support set of $\boldsymbol{\xi}$.

One can first estimate the nominal distribution based on the limited information of the disruption scenario occurrence. The uncertain disturbance can then be added to describe the true distribution. For convenience, the procedure (see Figure 3) to construct the Polyhedral and Box ambiguity sets mentioned above is developed.

In the following, the details about each step of the above procedure are described.

Step 1: Calculate the disruption probability of each supplier. The noisy-OR technique can be exploited to quantify the probability of occurrence of disruption scenarios. The number of disruption states faced by supplier i is $2^{|E_i|}$. Let u_{il}, $l \in \{1, \cdots, 2^{|E_i|}\}$, be the lth state faced by supplier i. We use $\pi(u_{il})$ to denote the probability of state u_{il}, and $\pi(\mathrm{H}_i|u_{il})$ to represent the probability that supplier i fails to operate in state u_{il}. These probabilities ($\pi(u_{il})$, $\pi(\mathrm{H}_i|u_{il})$) are usually obtained through historical data, expert knowledge, or a combination thereof. The disruption probability π_i of supplier i is equal to the sum of $\pi(\mathrm{H}_i|u_{il})\pi(u_{il})$ for all states. That is,

$$\pi_i = \sum_{u_{il}} \pi(\mathrm{H}_i|u_{il})\pi(u_{il}). \tag{22}$$

Here, the case in which supplier i (denoted H_i) faces two disruption events (e_{i1}, e_{i2}) is used as an example to show the state. If a disruption occurs at supplier i, it corresponds to one of the following four states: (i) both events e_{i1} and e_{i2} do not occur; (ii) event e_{i1} occurs, and event e_{i2} does not occur; (iii) event e_{i1} does not occur, and event e_{i2} occurs; and (iv) both events e_{i1} and e_{i2} occur. These four states are shown in Table 2.

Readers who are interested in using the noisy-OR technique to quantify the probability of occurrence of disruption scenarios can further refer to Hosseini and Ivanov (2019), Hosseini and Ivanov (2020), and Hosseini, Ivanov, and Blackhurst (2020).

Step 2: Calculate the nominal probability of occurrence for disruption scenario s. In scenario s, each supplier can either continue to operate or fail, and the set of all suppliers consists of two parts: the set of suppliers that continue to operate and the set of suppliers that fail to operate (i.e. disrupt). In scenario s, $I_s \cup \bar{I}_s = I$. Therefore, the nominal probability of occurrence of disruption scenario s can be calculated via the following equation:

$$\overline{\mathrm{P}}_s^0 = \prod_{i \in I_s}(1 - \pi_i) \times \prod_{i \in \bar{I}_s} \pi_i, \tag{23}$$

where $\overline{\mathrm{P}}_s^0$ denotes the nominal probability of occurrence for disruption scenario s.

Step 3: Select the first N scenarios with the highest likelihoods. According to the above process, there are $2^{|I|}$ possible disruption scenarios. Therefore, when the cardinality of the set I is large, the number of scenarios is large. In this paper, the scenario reduction method is employed to select the first N scenarios with the highest likelihood of scenarios of interest.

Remark 4.1: The decision-maker can also directly select the scenarios that he or she is more concerned about without depending on these probabilities. For example, the scenarios in which the probability of occurrence is small but the capacity of the provider drops significantly can also be selected by the decision-maker. In this paper, decision-makers are more concerned about those scenarios that have a comparatively higher probability of occurrence, and they rank these scenarios according to their probability of occurrence.

Step 4: Probability normalisation. The sum of the probabilities of the first N scenarios with the highest likelihoods is not 1. To solve this issue, we normalise this set

Table 2. Disruption states faced by supplier i.

Disruption events	State u_l	$\pi(\mathrm{H}_i\mid u)$	$\pi(u)$
e_{i1}	$u_1=\{\bar{e}_{i1},\bar{e}_{i2}\}$	$\pi(\mathrm{H}_i)$	$(1-\pi_{e_{i1}})\cdot(1-\pi_{e_{i2}})$
	$u_2=\{\bar{e}_{i1},e_{i2}\}$	$1-(1-\pi(\mathrm{H}_i))\cdot(1-\pi(\mathrm{H}_i\mid e_{i2}))$	$(1-\pi_{e_{i1}})\cdot\pi_{e_{i2}}$
e_{i2}	$u_3=\{e_{i1},\bar{e}_{i2}\}$	$1-(1-\pi(\mathrm{H}_i))\cdot(1-\pi(\mathrm{H}_i\mid e_{i1}))$	$\pi_{e_{i1}}\cdot(1-\pi_{e_{i2}})$
	$u_4=\{e_{i1},e_{i2}\}$	$1-(1-\pi(\mathrm{H}_i))\cdot(1-\pi(\mathrm{H}_i\mid e_{i1}))\cdot(1-\pi(\mathrm{H}_i\mid e_{i2}))$	$\pi_{e_{i1}}\cdot\pi_{e_{i2}}$

Note: $\{e_{i1},\bar{e}_{i2}\}$ denotes that event e_{i1} occurs and event e_{i2} doesn't occur. π (H_i) represents the probability that supplier i is disrupted but not due to e_{i1} and e_{i2}. π $(\mathrm{H}_i\mid e_{in})$ expresses the probability that disruptive event e_{in}causes disruption at supplier i.

of probabilities via the following normalisation equation:

$$\mathrm{P}_s^0=\frac{\overline{\mathrm{P}}_s^0}{\sum_{s=1}^{N}\overline{\mathrm{P}}_s^0}, \tag{24}$$

where P_s^0 denotes the final nominal probability of the selected sth scenario.

Step 5: Determine appropriate scales of ambiguity sets. We set $\mathbf{A}=\delta\cdot\mathrm{UM}$ (UM denotes unit matrix) in the Polyhedral ambiguity set and set $\boldsymbol{\xi}_U=-\boldsymbol{\xi}_L=\sigma\cdot\mathbf{P}_0$ in the Box ambiguity set. In general, δ and σ take values from $(0,1)$. When the scale parameters $\delta,\sigma\to 1$, the range of ambiguity sets becomes larger. The decision-maker can flexibly control the range of ambiguity sets by adjusting δ and σ according to his or her preferences. As a result, the developed Polyhedral and Box ambiguity sets have flexible structures.

In this subsection, two ambiguity sets have been demonstrated and a procedure is developed to construct them. In the next subsection, the computable form of Model (19) based on the constructed ambiguity sets is searched for.

4.3. Deriving the tractable robust counterparts

Thus far, we have shown how to construct the Polyhedral and Box ambiguity sets to characterise the uncertainty. The question is now how to derive the computable form of Model (19) based on the constructed ambiguity sets. For this purpose, we need to derive the computable form of the following maximisation problem:

$$\begin{aligned}&\max_{\mathbf{P}\in\mathcal{P}}\left\{\alpha\sum_{s\in S}\mathrm{P}_s\mathrm{TC}_{2,s}(\boldsymbol{q}^2(\mathrm{s}),\boldsymbol{q}^3(\mathrm{s}),\boldsymbol{z}(\mathrm{s}))\right.\\&\left.\quad+(1-\alpha)\left\{\phi+\frac{1}{1-\varepsilon}\sum_{s\in S}\mathrm{P}_s\mathrm{t}_s\right\}\right\}\\&=\max_{\mathbf{P}\in\mathcal{P}}\left\{\alpha\,\mathrm{TC}_2^{\mathrm{T}}\,\mathbf{P}\right.\\&\left.\quad+(1-\alpha)\left\{\phi+\frac{1}{1-\varepsilon}\mathbf{t}^{\mathrm{T}}\mathbf{P}\right\}\right\},\end{aligned} \tag{25}$$

where $\mathrm{TC}_2=(\mathrm{TC}_{2,s}(\boldsymbol{q}^2(s),\boldsymbol{q}^3(s),\boldsymbol{z}(s)))_{s\in S}$ and $\mathbf{t}=(\mathrm{t}_s)_{s\in S}$.

On the one hand, under Polyhedral ambiguity set (20), the computationally tractable form of Model (19) is derived by the following theorems.

Theorem 4.1: *Under ambiguity set* (20), *maximisation problem* (25) *is equivalent to the following minimisation problem:*

$$\begin{aligned}&min\quad\alpha(TC_2^T\boldsymbol{P}_0+\boldsymbol{P}_0^T\boldsymbol{\Psi}+\theta)\\&\quad+(1-\alpha)\left\{\phi+\frac{\boldsymbol{t}^T\boldsymbol{P}_0+\boldsymbol{P}_0^T\hat{\boldsymbol{\Psi}}+\hat{\theta}}{1-\varepsilon}\right\}\end{aligned}$$

$$s.t.\quad\|\boldsymbol{A}^T\,TC_2+\boldsymbol{A}^T\boldsymbol{\Psi}-\boldsymbol{A}^T\boldsymbol{e}\mu\|_*\le\theta, \tag{26}$$

$$\boldsymbol{\Psi}\ge 0,\theta\ge 0, \tag{27}$$

$$\|\boldsymbol{A}^T\boldsymbol{t}+\boldsymbol{A}^T\hat{\boldsymbol{\Psi}}-\boldsymbol{A}^T\boldsymbol{e}\hat{\mu}\|_*\le\hat{\theta}, \tag{28}$$

$$\hat{\boldsymbol{\Psi}}\ge 0,\hat{\theta}\ge 0, \tag{29}$$

where θ, $\boldsymbol{\Psi}$, μ, $\hat{\theta}$, $\hat{\boldsymbol{\Psi}}$, *and* $\hat{\mu}$ *are decision variables, and* $\|\cdot\|_*=\|\cdot\|_\infty$ *is the dual of* $\|\cdot\|_1$.

Proof: The proof of Theorem 1 is in Appendix 2.

On the other hand, under Box ambiguity set (21), the computationally tractable form of Model (19) is derived by the following theorem. ■

Theorem 4.2: *Based on ambiguity set* (21), *maximisation problem* (25) *is equivalent to the following minimisation problem:*

$$\begin{aligned}&\min\quad\alpha(TC_2^T\boldsymbol{P}_0+\boldsymbol{\xi}_U^T\boldsymbol{\pi}-\boldsymbol{\xi}_L^T\boldsymbol{\gamma})\\&\quad+(1-\alpha)\left\{\phi+\frac{\boldsymbol{t}^T\boldsymbol{P}_0+\boldsymbol{\xi}_U^T\hat{\boldsymbol{\pi}}-\boldsymbol{\xi}_L^T\hat{\boldsymbol{\gamma}}}{1-\varepsilon}\right\}\end{aligned}$$

$$s.t.\quad\boldsymbol{e}\tau+\boldsymbol{\pi}-\boldsymbol{\gamma}=TC_2, \tag{30}$$

$$\boldsymbol{\pi}\ge 0,\boldsymbol{\gamma}\ge 0, \tag{31}$$

$$\boldsymbol{e}\hat{\tau}+\hat{\boldsymbol{\pi}}-\hat{\boldsymbol{\gamma}}=\boldsymbol{t}, \tag{32}$$

$$\hat{\boldsymbol{\pi}}\ge 0,\hat{\boldsymbol{\gamma}}\ge 0, \tag{33}$$

where τ, $\boldsymbol{\pi}$, $\boldsymbol{\gamma}$, $\hat{\tau}$, $\hat{\boldsymbol{\pi}}$, *and* $\hat{\boldsymbol{\gamma}}$ *are decision variables.*

Proof: The proof of Theorem 2 is in Appendix 3. ■

4.4. Computationally tractable reformulations for Model (19)

First, the multiplications of two binary variables defined in constraint (1), e.g.$x_i x_j$, are nonlinear. Proposition 1 and Corollary 1, which are presented in Appendix 1, linearise constraint (1) to constraints (A1)–(A11).

Second, by applying Theorem 1, Model (19) is represented as

$$\text{Polyhedral}: \begin{cases} \min\limits_{\varphi'} & \mathrm{TC}_1(\boldsymbol{x},\boldsymbol{x}',\boldsymbol{q}^1)+\alpha(\mathrm{TC}_2^{\mathrm{T}}\mathbf{P}_0 \\ & +\mathbf{P}_0^{\mathrm{T}}\boldsymbol{\Psi}+\theta) \\ & +(1-\alpha)\left\{\phi+\dfrac{\mathbf{t}^{\mathrm{T}}\mathbf{P}_0+\mathbf{P}_0^{\mathrm{T}}\hat{\boldsymbol{\Psi}}+\hat{\theta}}{1-\varepsilon}\right\} \\ \text{s.t.} & \text{Constraints (2)-(18),(26)-(29),} \\ & \text{(A1)-(A11),} \end{cases}$$

where $\varphi' = (\varphi, \theta, \boldsymbol{\Psi}, \mu, \hat{\theta}, \hat{\boldsymbol{\Psi}}, \hat{\mu})$.

Third, by applying Theorem 2, Model (19) is represented as

$$\text{Box}: \begin{cases} \min\limits_{\varphi''} & \mathrm{TC}_1(\boldsymbol{x},\boldsymbol{x}',\boldsymbol{q}^1) \\ & +\alpha(\mathrm{TC}_2^{\mathrm{T}}\mathbf{P}_0+\boldsymbol{\xi}_U^{\mathrm{T}}\boldsymbol{\pi}-\boldsymbol{\xi}_L^{\mathrm{T}}\boldsymbol{\gamma}) \\ & +(1-\alpha)\left\{\phi+\dfrac{\mathbf{t}^{\mathrm{T}}\mathbf{P}_0+\boldsymbol{\xi}_U^{\mathrm{T}}\hat{\boldsymbol{\pi}}-\boldsymbol{\xi}_L^{\mathrm{T}}\hat{\boldsymbol{\gamma}}}{1-\varepsilon}\right\} \\ \text{s.t.} & \text{Constraints (2)-(18), (30)-(33),} \\ & \text{(A1)-(A11),} \end{cases}$$

where $\varphi'' = (\varphi, \tau, \boldsymbol{\pi}, \boldsymbol{\gamma}, \hat{\tau}, \hat{\boldsymbol{\pi}}, \hat{\boldsymbol{\gamma}})$.

The above Polyhedral and Box models are deterministic MILPs, which can be solved efficiently by general commercial software. In the following section, we will illustrate the validity of the proposed optimisation approach via a case study.

5. Case study

A case study is used to conduct some numerical experiments to show the feasibility and effectiveness of the proposed two-stage DR Mean-CVaR model. All numerical experiments are solved by CPLEX 12.8.0 optimisation software on an Inter(R) Core(TM) i7-6500U 2.50 GHz personal computer with 8 GB RAM operating under Windows 10 (64 bit).

5.1. Problem background and data source

Huawei is a famous technology company that manufactures electronic devices. Its head office is located in Shenzhen, Guangdong Province, China. Because of the huge market demand for Huawei's equipment, its own production lines are not up to the tremendous production task. As a result, Huawei outsources many of its cell phone assembly tasks to other plants, such as the enterprise BYD, which is located in Changsha, Hunan Province. To date, most Huawei cell phones on the market have been assembled by BYD[1]. Typically, the components of Huawei cell phones are provided by multiple suppliers. By doing so, it can satisfy the huge demand in a short period and can also increase the resilience of the supply chain. Taking the memory chip of cell phones, Huawei has established cooperative relationships with three suppliers: Samsung (located in Xian), Micron (located in Xian), and Hynix (located in Wuxi). At the same time, Huawei also has connections with Beijing-Xicheng (located in Beijing), GigaDevice (located in Shanghai), Yangtze (located in Wuhan), and CXMT (located in Hefei)[2].

The memory chip of Huawei cell phones is used as the product for this case and Hynix (H_1), Micron (H_2), GigaDevice (H_3), Yangtze (H_4) and CXMT (H_5) are considered as potential suppliers. The Changsha BYD (Huawei's assembly base) is the manufacturer. The locations of the suppliers and manufacturer are shown in Figure 4. In the BYD assembly plants, the memory chips of these cell phones may come from several of these five suppliers. After these cell phones are processed at BYD, they are shipped to warehouses across the country and then sold to customers. The decision-maker needs to identify the main suppliers, the backup suppliers, as well as the quantity of products from these suppliers.

According to the IDC Worldwide Quarterly Mobile Phone Tracker[3], the shipment quantity of Huawei cell phones is approximately $21,700,000$ in 2020Q4. Here, we take this shipment quantity as demand. Therefore, $D = 21,700,000$ We set the allowed maximum number of main suppliers $R = 2$. Since $C_i' \geq C_i$ and $L_i'' \geq L_i' \geq L_i$, without loss of generality, we let C_i' equal C_i+Uniform($5 \cdot 10^5$, $10 \cdot 10^5$), L_i' equal L_i+ Uniform(3, 5), and L_i'' equal L_i+ Uniform(1, 3). In addition, according to the actual situation, we first set the least segregation distance $Sd = 300$ km and the least total segregation distance $TD = 2000$ km. In the experiments, the sensitivity analysis for Sd and TD is performed. Regarding the costs in Table 3, we refer to the mentioned websites and some relevant literature (e.g. Torabi, Baghersad, and Mansouri (2015), Vahidi, Torabi, and Ramezankhani (2018)) and then reasonably set the values of these parameters. The locations and all distances (see Table 4) are obtained via Baidu Maps[4]. The percentages θ_{is}, $\forall i \in I, s \in S$ are uniformly generated from the interval (0.6, 0.8).

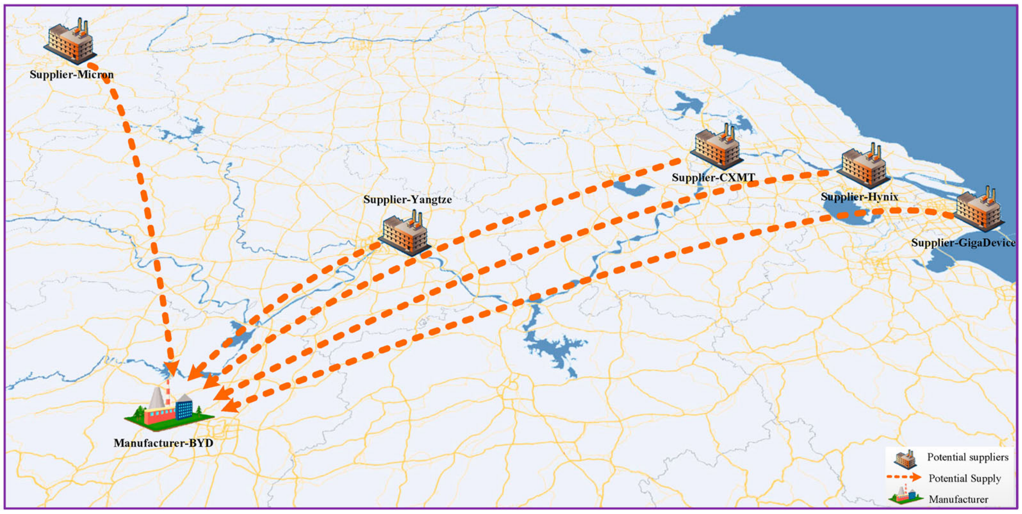


Figure 4. The locations of the suppliers and the manufacturer.

Table 3. The values of some parameters and their generation ranges.

Parameter	H_1	H_2	H_3	H_4	H_5	Generation range
C_i	2,370,705	2,559,845	2,973,594	2,845,478	2,733,249	Uniform $(2 \cdot 10^6, 3 \cdot 10^6)$
C_i'	2,969,846	3,346,248	3,656,969	3,372,341	3,290,102	C_i+Uniform $(5 \cdot 10^5, 10 \cdot 10^5)$
L_i	17.4	17.6	18.1	17.9	15.5	Uniform $(15, 20)$
L_i'	21	21.8	21.7	21.5	20	L_i+Uniform $(3, 5)$
L_i''	18.5	18.7	19.6	19.6	18.1	L_i+Uniform $(1, 3)$
Ca_i	15,256,617	14,570,328	9,237,704	15,053,244	11,023,838	Uniform $(9 \cdot 10^6, 1.6 \cdot 10^7)$

Table 4. The distances (km) between suppliers.

	H_1	H_2	H_3	H_4	H_5
H_1	0	1116.9	120.4	564.3	321.4
H_2	1116.9	0	1237.3	680.2	801
H_3	120.4	1237.3	0	673	441.3
H_4	564.3	680.2	673	0	284.6
H_5	321.4	801	441.3	284.6	0

5.2. Constructing ambiguity sets

Now, the Polyhedral and Box ambiguity sets are constructed based on the procedure proposed in subsection 4.2. In this case study, there are 5 suppliers and 32 scenarios. It is assumed that each supplier faces two disruption events. The sets E_i for H_1, H_2, and H_5 are Fire, Earthquake, and the sets E_i for H_3 and H_4 are Hurricane, Flood. In the first step to construct the ambiguity set, we obtain that disruption probabilities $(\pi_i)_{i\in[5]}$ of suppliers are 0.268, 0.242, 0.19, 0.187, and 0.274. Based on $(\pi_i)_{i\in[5]}$, we directly calculate $\overline{\mathrm{P}}_s^0$ by the second step. Here, the following 5 groups of scenarios are selected: $N = 12$, $N = 15$, $N = 18$, $N = 21$ and $N = 24$. After probability normalisation, the obtained nominal probabilities of occurrence for the selected scenarios are listed in Table 5. In addition, without loss of generality, we set $\delta = 0.2$ and $\sigma = 0.3$.

After constructing the Polyhedral and Box ambiguity sets for our case study, the following experiments are first carried out based on $N = 15$. The effect of changing the number of selected scenarios on the results of the models is explored.

5.3. Computational results of the proposed model

In this subsection, the computational results for the Polyhedral and Box models are reported.

The optimal value determined by the Polyhedral model with $\delta = 0.2, \alpha = 0.5, \varepsilon = 0.9$ is 376512212.51. The Polyhedral model selects 3 suppliers to cooperate with. They are H_1, H_2 and H_5, where H_2 and H_5 are chosen as the main suppliers and H_1 is selected as the backup supplier. The total distance among all selected suppliers is 2239.3km. The optimal order allocation strategy is as follows. In the first stage, the quantity of products contracted with H_2 is $1.122 \cdot 10^7$ and with H_5 is $1.048 \cdot 10^7$. In the second stage, H_1 delivers products to the manufacturer in some scenarios, such as the quantity of delivering products is $2.949 \cdot 10^5$ in the second scenario; the main suppliers H_2 and H_5 deliver surplus products in some scenarios, such as the surplus quantity of H_2 is $2.323 \cdot 10^6$ in the seventh scenario, and the surplus quantity of H_5 is $1.457 \cdot 10^5$ in the ninth scenario. Table 6 shows

Table 5. Normalised nominal probabilities for the selected scenarios.

Scenario		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}
Nominal probability P_0	$N = 12$	0.3160	0.1193	0.1157	0.1009	0.0741	0.0727	0.0437	0.0381	0.0369	0.0280	0.0274	0.0271
	$N = 15$	0.2944	0.1111	0.1078	0.0940	0.0691	0.0677	0.0407	0.0355	0.0344	0.0261	0.0256	0.0253
	$N = 18$	0.2835	0.1070	0.1038	0.0905	0.0665	0.0652	0.0392	0.0342	0.0331	0.0251	0.0246	0.0243
	$N = 21$	0.2766	0.1044	0.1013	0.0883	0.0649	0.0636	0.0382	0.0333	0.0323	0.0245	0.0240	0.0238
	$N = 24$	0.2710	0.1023	0.0992	0.0865	0.0636	0.0623	0.0375	0.0327	0.0317	0.0240	0.0235	0.0233

Scenario		s_{13}	s_{14}	s_{15}	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}	s_{21}	s_{22}	s_{23}	s_{24}
Nominal probability P_0	$N = 12$	—	—	—	—	—	—	—	—	—	—	—	—
	$N = 15$	0.0248	0.0220	0.0216	—	—	—	—	—	—	—	—	—
	$N = 18$	0.0239	0.0212	0.0208	0.0153	0.0125	0.0092	—	—	—	—	—	—
	$N = 21$	0.0233	0.0207	0.0203	0.0149	0.0122	0.0090	0.0088	0.0078	0.0077	—	—	—
	$N = 24$	0.0228	0.0203	0.0199	0.0146	0.0120	0.0088	0.0086	0.0077	0.0075	0.0074	0.0073	0.0055

the optimal SS&OA scheme provided by the Polyhedral model.

The optimal value determined by the Box model with $\sigma = 0.3, \alpha = 0.5, \varepsilon = 0.9$ is $376, 358, 734.37$, which is slightly lower than the optimal objective provided by the Polyhedral model. Similarly, the Box model selects H_1, H_2 and H_5 to cooperate with, where H_2 and H_5 are selected as the main suppliers and H_1 is selected as the backup supplier. The optimal order allocation tactic is as follows. In the first stage, the quantity of orders contracted with H_2 is $1.107 \cdot 10^7$ and with H_5 is $1.063 \cdot 10^7$. In the second stage, H_1 delivers products to the manufacturer in some scenarios. For example, the quantity of delivering products is $2.028 \cdot 10^6$ in the eighth scenario; H_2 and H_5 deliver surplus products in some scenarios. For example, the surplus quantity of H_2 is $3.24 \cdot 10^6$ in the eleventh scenario, and the surplus quantity of H_5 is $3.973 \cdot 10^5$ in the fifteenth scenario. The optimal SS&OA strategy identified by the Box model is shown in Table 7.

5.4. Comparison with the nominal model

When Sawik (2013b), Torabi, Baghersad, and Mansouri (2015), Ni, Howell, and Sharkey (2018), and Hosseini et al. (2019b) studied the SS&OA problem, they all assumed that the probabilities of occurrences for disruption scenarios are deterministic. Now, we ignore the probability uncertainty of occurrence for disruption scenarios, i.e. these probabilities take their nominal values. In this case, the proposed two-stage DR Mean-CVaR model degenerates into a two-stage stochastic Mean-CVaR model, which is our nominal model. The solution provided by the nominal model tends to yield a weak robust SS&OA scheme because it ignores the uncertainties of the probabilities. The following experimental results of the nominal model support these arguments.

The optimal value identified by the nominal model with $\alpha = 0.5, \varepsilon = 0.9$ is $375, 786, 400.44$, which is slightly lower than the optimal objectives provided by the Polyhedral and Box models. The nominal model also selects three suppliers H_1, H_2 and H_5 to cooperate with. Different from the Polyhedral and Box models, suppliers H_1 and H_5 are selected as the main suppliers, and H_2 is selected as the backup supplier by the nominal model. In the first stage, the quantity of products contracted with H_1 is $1.068 \cdot 10^7$ and with H_5 is $1.102 \cdot 10^7$. In the second stage, H_2 delivers products to the manufacturer in some scenarios. For example, the quantity of delivering products is $1.217 \cdot 10^6$ in the third scenario. H_1 delivers surplus products in some scenarios. For example, the surplus quantity is $4.189 \cdot 10^6$ in the second scenario. However, the main supplier H_5 does not deliver surplus products in all scenarios. Table 8 displays in detail the optimal SS&OA scheme offered by the nominal model.

By comparing the experimental results of the Polyhedral model, Box model, and nominal model, it can be concluded that the uncertainties of the probabilities of occurrence for disruption scenarios have an obvious impact on the optimal SS&OA scheme. To further compare the two-stage DR Mean-CVaR model with its nominal model, following the idea of Ma, Liu, and Liu (2020), we introduce the distributionally robust price (PDR),

$$\mathrm{PDR} = (\mathrm{DR})^* - (\mathrm{Nominal})^*,$$

where $(\cdot)^*$ denotes the optimal cost.

The PDR represents the extra cost of the optimal scheme provided by the DR model to immunise the influence caused by the uncertain probabilities of disruption scenario occurrence compared with the nominal model. After calculation, the PDR of the Polyhedral model with $\delta = 0.2, \alpha = 0.5, \varepsilon = 0.9$ is $725, 812.07$, and the PDR of the Box model with $\sigma = 0.3, \alpha = 0.5, \varepsilon = 0.9$ is $572, 342.93$. The former is higher than the latter. That is, the extra cost corresponding to the Polyhedral model to resist the uncertainty of probabilities is higher than that corresponding to the Box model.

Table 6. The optimal SS&OA scheme identified by the Polyhedral model with $\delta = 0.2, \alpha = 0.5, \varepsilon = 0.9$.

Obj	376,512,212.51														
Main supplier	H_2	q_2: $1.122K_1$													
	$q_{2,1}$	$q_{2,2}$	$q_{2,3}$	$q_{2,4}$	$q_{2,5}$	$q_{2,6}$	$q_{2,7}$	$q_{2,8}$	$q_{2,9}$	$q_{2,10}$	$q_{2,11}$	$q_{2,12}$	$q_{2,13}$	$q_{2,14}$	$q_{2,15}$
	$1.122K_1$	$1.122K_1$	$1.122K_1$	$9.762K_2$	$1.122K_1$	$1.122K_1$	$1.122K_1$	$1.122K_1$	$1.122K_1$	$1.122K_1$	$1.122K_1$	$1.122K_1$	$1.122K_1$	$9.033K_2$	$9.325K_2$
	$z_{2,1}$	$z_{2,2}$	$z_{2,3}$	$z_{2,4}$	$z_{2,5}$	$z_{2,6}$	$z_{2,7}$	$z_{2,8}$	$z_{2,9}$	$z_{2,10}$	$z_{2,11}$	$z_{2,12}$	$z_{2,13}$	$z_{2,14}$	$z_{2,15}$
	0	$3.512K_2$	0	0	0	0	$2.323K_2$	0	0	$2.984K_2$	$3.095K_2$	0	0	0	0
	H_5	q_5: $1.048K_1$													
	$q_{5,1}$	$q_{5,2}$	$q_{5,3}$	$q_{5,4}$	$q_{5,5}$	$q_{5,6}$	$q_{5,7}$	$q_{5,8}$	$q_{5,9}$	$q_{5,10}$	$q_{5,11}$	$q_{5,12}$	$q_{5,13}$	$q_{5,14}$	$q_{5,15}$
	$1.048K_1$	$6.835K_2$	$1.048K_1$	$1.048K_1$	$1.048K_1$	$1.048K_1$	$8.157K_2$	$8.598K_2$	$1.048K_1$	$7.496K_2$	$7.386K_2$	$1.048K_1$	$1.048K_1$	$1.048K_1$	$1.048K_1$
	$z_{5,1}$	$z_{5,2}$	$z_{5,3}$	$z_{5,4}$	$z_{5,5}$	$z_{5,6}$	$z_{5,7}$	$z_{5,8}$	$z_{5,9}$	$z_{5,10}$	$z_{5,11}$	$z_{5,12}$	$z_{5,13}$	$z_{5,14}$	$z_{5,15}$
	0	0	0	$5.43K_3$	0	0	0	0	$1.457K_3$	0	0	0	0	$5.43K_3$	$5.43K_3$
Backup supplier	H_1														
	$q'_{1,1}$	$q'_{1,2}$	$q'_{1,3}$	$q'_{1,4}$	$q'_{1,5}$	$q'_{1,6}$	$q'_{1,7}$	$q'_{1,8}$	$q'_{1,9}$	$q'_{1,10}$	$q'_{1,11}$	$q'_{1,12}$	$q'_{1,13}$	$q'_{1,14}$	$q'_{1,15}$
	0	$2.949K_3$	0	$9.14K_3$	0	0	0	$1.882K_2$	0	0	0	0	0	$1.6426K_2$	$1.3512K_2$

Note: Obj denotes objective, K_1 denotes 10^7, K_2 denotes 10^6, K_3 denotes 10^5.

Table 7. The optimal SS&OA scheme identified by the Box model with $\sigma = 0.3, \alpha = 0.5, \varepsilon = 0.9$.

Obj	376,358,734.37														
Main supplier	H_2	q_2: $1.107K_1$													
	$q_{2,1}$	$q_{2,2}$	$q_{2,3}$	$q_{2,4}$	$q_{2,5}$	$q_{2,6}$	$q_{2,7}$	$q_{2,8}$	$q_{2,9}$	$q_{2,10}$	$q_{2,11}$	$q_{2,12}$	$q_{2,13}$	$q_{2,14}$	$q_{2,15}$
	$1.107K_1$	$1.107K_1$	$1.107K_1$	$9.762K_2$	$1.107K_1$	$1.107K_1$	$1.107K_1$	$1.107K_1$	$1.107K_1$	$1.107K_1$	$1.107K_1$	$1.107K_1$	$1.107K_1$	$9.033K_2$	$9.325K_2$
	$z_{2,1}$	$z_{2,2}$	$z_{2,3}$	$z_{2,4}$	$z_{2,5}$	$z_{2,6}$	$z_{2,7}$	$z_{2,8}$	$z_{2,9}$	$z_{2,10}$	$z_{2,11}$	$z_{2,12}$	$z_{2,13}$	$z_{2,14}$	$z_{2,15}$
	0	$3.497K_2$	0	0	0	0	$2.469K_2$	0	0	$3.13K_2$	$3.24K_2$	0	0	0	0
	H_5	q_5: $1.063K_1$													
	$q_{5,1}$	$q_{5,2}$	$q_{5,3}$	$q_{5,4}$	$q_{5,5}$	$q_{5,6}$	$q_{5,7}$	$q_{5,8}$	$q_{5,9}$	$q_{5,10}$	$q_{5,11}$	$q_{5,12}$	$q_{5,13}$	$q_{5,14}$	$q_{5,15}$
	$1.063K_1$	$6.835K_2$	$1.063K_1$	$1.063K_1$	$1.063K_1$	$1.063K_1$	$8.157K_2$	$8.598K_2$	$1.063K_1$	$7.496K_2$	$7.386K_2$	$1.063K_1$	$1.063K_1$	$1.063K_1$	$1.063K_1$
	$z_{5,1}$	$z_{5,2}$	$z_{5,3}$	$z_{5,4}$	$z_{5,5}$	$z_{5,6}$	$z_{5,7}$	$z_{5,8}$	$z_{5,9}$	$z_{5,10}$	$z_{5,11}$	$z_{5,12}$	$z_{5,13}$	$z_{5,14}$	$z_{5,15}$
	0	0	0	$3.973K_3$	0	0	0	0	0	0	0	0	0	$3.973K_3$	$3.973K_3$
Backup supplier	H_1														
	$q'_{1,1}$	$q'_{1,2}$	$q'_{1,3}$	$q'_{1,4}$	$q'_{1,5}$	$q'_{1,6}$	$q'_{1,7}$	$q'_{1,8}$	$q'_{1,9}$	$q'_{1,10}$	$q'_{1,11}$	$q'_{1,12}$	$q'_{1,13}$	$q'_{1,14}$	$q'_{1,15}$
	0	$2.949K_3$	0	$9.14K_3$	0	0	0	$2.028K_2$	0	0	0	0	0	$1.6426K_2$	$1.3512K_2$

Note: Obj denotes objective, K_1 denotes 10^7, K_2 denotes 10^6, K_3 denotes 10^5.

Table 8. The optimal SS&OA scheme identified by the nominal model with $\alpha = 0.5, \varepsilon = 0.9$.

Obj	375,786,400.44														
Main supplier	H_1	q_1: $1.068K_1$													
	$q_{1,1}$	$q_{1,2}$	$q_{1,3}$	$q_{1,4}$	$q_{1,5}$	$q_{1,6}$	$q_{1,7}$	$q_{1,8}$	$q_{1,9}$	$q_{1,10}$	$q_{1,11}$	$q_{1,12}$	$q_{1,13}$	$q_{1,14}$	$q_{1,15}$
	$1.068K_1$	$1.068K_1$	$9.459K_2$	$1.068K_1$	$1.068K_1$	$1.068K_1$	$9.154K_2$	$1.068K_1$	$1.068K_1$	$1.068K_1$	$1.068K_1$	$9.917K_2$	$1.068K_1$	$1.068K_1$	$1.068K_1$
	$z_{1,1}$	$z_{1,2}$	$z_{1,3}$	$z_{1,4}$	$z_{1,5}$	$z_{1,6}$	$z_{1,7}$	$z_{1,8}$	$z_{1,9}$	$z_{1,10}$	$z_{1,11}$	$z_{1,12}$	$z_{1,13}$	$z_{1,14}$	$z_{1,15}$
	0	$4.189K_2$	0	0	0	0	0	$2.425K_2$	0	$3.527K_2$	$3.638K_2$	0	0	0	0
	H_5	q_5: $1.102K_1$													
	$q_{5,1}$	$q_{5,2}$	$q_{5,3}$	$q_{5,4}$	$q_{5,5}$	$q_{5,6}$	$q_{5,7}$	$q_{5,8}$	$q_{5,9}$	$q_{5,10}$	$q_{5,11}$	$q_{5,12}$	$q_{5,13}$	$q_{5,14}$	$q_{5,15}$
	$1.102K_1$	$6.835K_2$	$1.102K_1$	$1.102K_1$	$1.102K_1$	$1.102K_1$	$8.157K_2$	$8.598K_2$	$1.102K_1$	$7.496K_2$	$7.386K_2$	$1.102K_1$	$1.102K_1$	$1.102K_1$	$1.102K_1$
	$z_{5,1}$	$z_{5,2}$	$z_{5,3}$	$z_{5,4}$	$z_{5,5}$	$z_{5,6}$	$z_{5,7}$	$z_{5,8}$	$z_{5,9}$	$z_{5,10}$	$z_{5,11}$	$z_{5,12}$	$z_{5,13}$	$z_{5,14}$	$z_{5,15}$
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Backup supplier	H_2														
	$q'_{2,1}$	$q'_{2,2}$	$q'_{2,3}$	$q'_{2,4}$	$q'_{2,5}$	$q'_{2,6}$	$q'_{2,7}$	$q'_{2,8}$	$q'_{2,9}$	$q'_{2,10}$	$q'_{2,11}$	$q'_{2,12}$	$q'_{2,13}$	$q'_{2,14}$	$q'_{2,15}$
	0	0	$1.217K_2$	0	0	0	$4.388K_2$	0	0	0	0	$7.593K_2$	0	0	0

Note: Obj denotes objective, K_1 denotes 10^7, K_2 denotes 10^6, K_3 denotes 10^5.

5.5. Effects about trade-off and confidence level parameters

In this subsection, to explore the effect of the changes in the trade-off parameter α and the confidence level parameter ε on the optimal objective and PDR, sensitivity analyses of α and ε are conducted. We set $\delta = 0.1$ for the Polyhedral model, $\sigma = 0.3$ for the Box model, α takes values from 0.2 to 0.8 in intervals of 0.1, and ε takes values from 0.75 to 0.95 in intervals of 0.05. Other parameters remain unchanged. The experimental results are displayed in Figures 5 and 6.

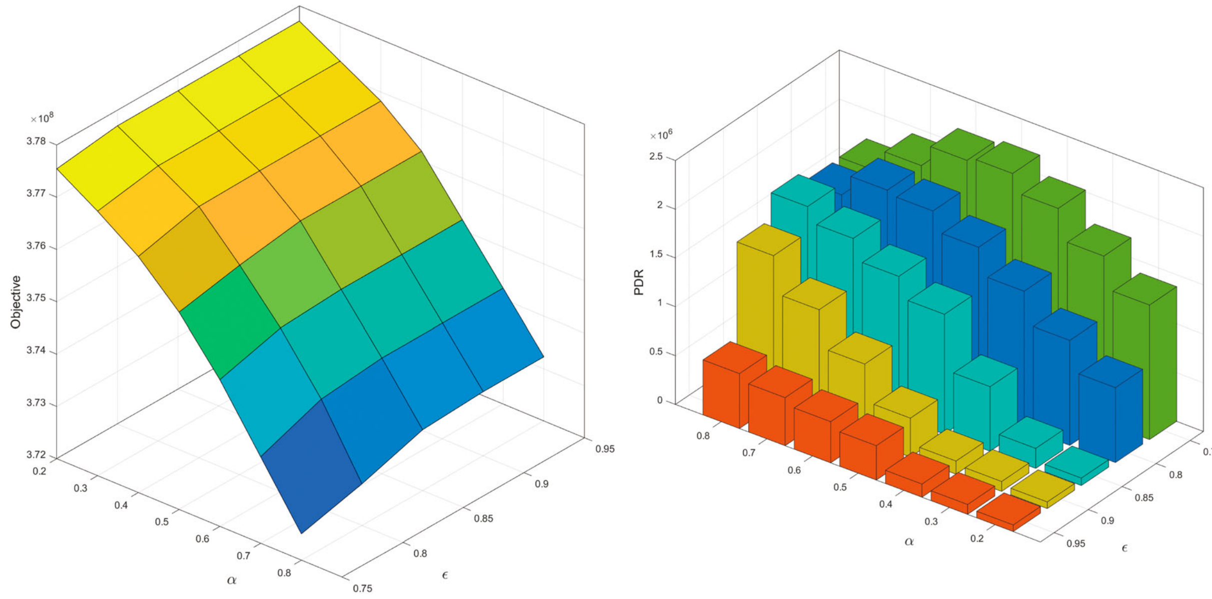
From Figure 5(a), when ε is fixed, the optimal value of the Polyhedral model decreases as α increases because the proportion of Mean increases and the proportion of CVaR decreases in the objective function. When α is fixed, the optimal value of the Polyhedral model increases as ε increases. According to Figure 5(b), when ε is stationary, the PDR of the Polyhedral model increases as α increases. That is, although the optimal objective decreases if α increases, the price increases. When α is fixed, the PDR of the Polyhedral model decreases as ε increases.

From Figure 6(a), when ε is stationary, we observe that the optimal objective of the Box model decreases as α increases. The reasons for this are the same as in the Polyhedral model case. According to Figure 6(b), when ε is fixed, the PDR of the Box model increases with α. When α is fixed, as ε increases, the optimal objective of the Box model increases, and the PDR decreases.

5.6. Effects about sizes of ambiguity sets

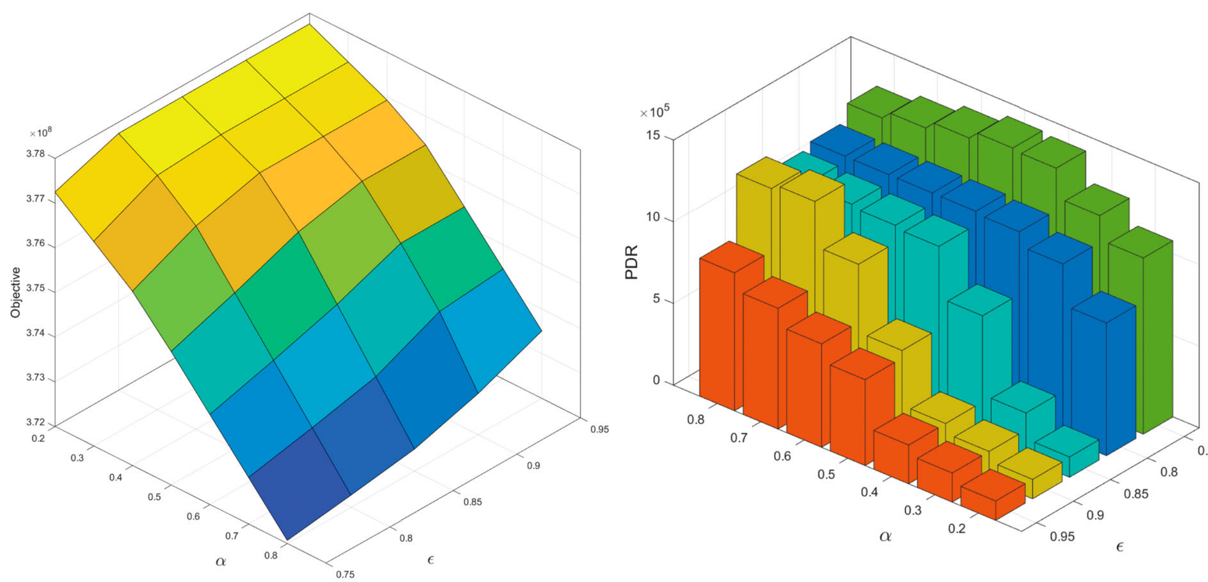
In this subsection, sensitivity analyses of δ and σ are conducted to explore the effect of the sizes of ambiguity sets on the optimal value and PDR. We set $\alpha = 0.5, \varepsilon = 0.9$ for the Polyhedral and Box models, δ takes values from 0.05 to 0.35 in intervals of 0.05, and σ takes values from 0.2 to 0.5 in intervals of 0.05. Other parameters remain unchanged. For the Polyhedral ambiguity set, if δ progressively increases, the size of the Polyhedral ambiguity set also increases. Similarly, as σ becomes larger, the size of the Box ambiguity set becomes larger. The experimental results are shown in Figure 7.

From Figure 7, it can be observed that the optimal objectives of the Polyhedral and Box models increase with δ and σ, respectively. At the same time, the increase in the optimal objectives of the above models leads to the increase in the PDRs of the Polyhedral and Box models. This phenomenon is rational. The reason is that as the sizes of the Polyhedral and Box ambiguity sets become progressively larger, the Polyhedral and Box models need more extra costs to immunise against the influence



(a) (b)

Figure 5. The objective and PDR of the Polyhedral model with $\delta = 0.1$ under different α and ε. (a) Objective (b) PDR.



(a) (b)

Figure 6. The objective and PDR of the Box model with $\sigma = 0.3$ under different α and ε. (a) Objective (b) PDR.

caused by the uncertain probability of occurrence for disruption scenarios. Therefore, the optimal objectives and PDRs of the Polyhedral and Box models increase with the size of the corresponding ambiguity set.

5.7. Effects about the number of selected scenarios

In this subsection, the effect of the number (N) of selected scenarios on the objectives of the Polyhedral and Box models is explored. We set N to the following values: 12, 15, 18, 21, and 24. The other parameters are set as in the previous subsection. The experimental results are reported in Figure 8 and Table 9.

From Figure 8, it can be observed that the optimal objectives of the Polyhedral and Box models increase with N. The more disruption scenarios the decision-maker selected mean that the optimal SS&OA schemes generated by the Polyhedral and Box models need to take into account more scenarios. From this observation, the objectives of the Polyhedral and Box models increase with N.

The number of selected scenarios affects the optimal SS&OA schemes provided by the Polyhedral and Box

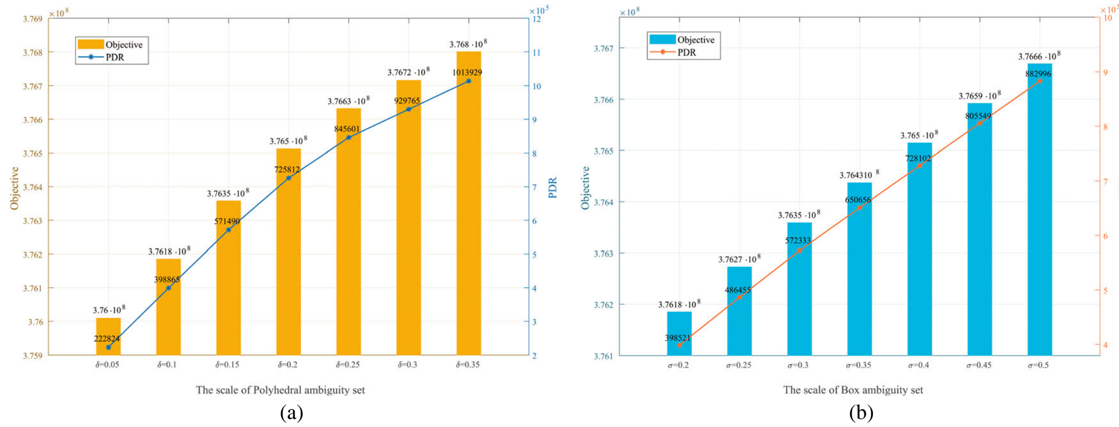


Figure 7. The objective and PDR under different δ and σ. (a) Polyhedral model with $a = 0.5$, $\varepsilon = 0.9$ (b) Box model with $a = 0.5$, $\varepsilon = 0.9$.

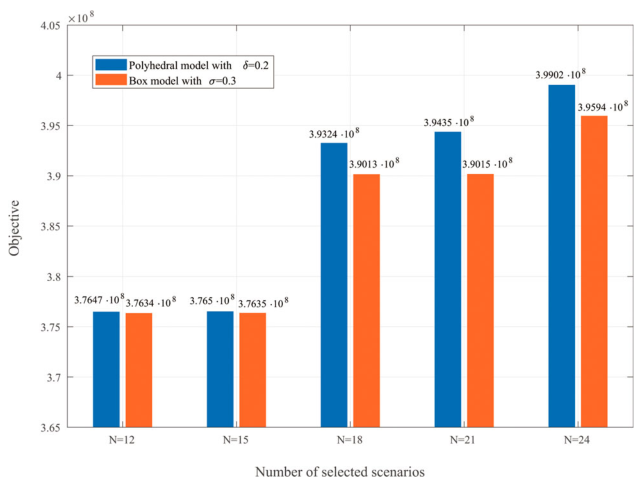


Figure 8. The objectives of models with $\alpha = 0.5, \varepsilon = 0.9$ under different N.

models. Table 9 provides the optimal SS&OA schemes obtained by the Polyhedral model with $\delta = 0.2, \alpha = 0.5$, $\varepsilon = 0.9$ and Box model with $\sigma = 0.3$, $\alpha = 0.5$, $\varepsilon = 0.9$ under $N = 24$. From Table 9, it is found that both models select H_2 and H_4 as main suppliers and H_1 as backup supplier. Supplier H_5 is not selected. In addition, the optimal order quantities in the pre-disruption and post-disruption stages also change. The details of the optimal SS&OA schemes identified by these two models are shown in Table 9.

5.8. Effects about resilience distances

In this subsection, sensitivity analyses of Sd and TD are performed. We set $\delta = 0.2, \alpha = 0.5, \varepsilon = 0.9$ for the Polyhedral model and $\sigma = 0.3, \alpha = 0.5, \varepsilon = 0.9$ for the Box model. We let (Sd, TD) take the following values:$(200, 2000)$, $(400, 2000)$, $(400, 2500)$, and $(200, 3000)$. Other parameters remain unchanged. The experimental results are provided in Table 10 and Figure 9.

From Table 10, it is found that as TD increases, the number of suppliers selected for cooperation increases. For example, three suppliers are selected for cooperation when $TD < 3000$, and four suppliers are selected when $TD = 3000$. This observed phenomenon is justified. The decision-maker should cooperate with more suppliers to meet the resilience distance requirement when the required least total segregation distance among all selected suppliers increases. In addition, for Sd and TD, when one of these two parameters is constant, the total distance among the selected suppliers (i.e. the resilience of the supply chain) increases with the other parameter.

From Figure 9, when Sd is constant, for the Polyhedral and Box models, the optimal objectives increase with TD. This is mainly because when TD becomes larger, the total distance among the selected suppliers becomes larger, which increases the cooperation cost and product's transportation cost. When TD is constant, the objectives of these two models also increase as Sd increases. The reason is mainly that the total distance among the selected suppliers increases with Sd, which increases the product's transportation cost.

6. Managerial insights

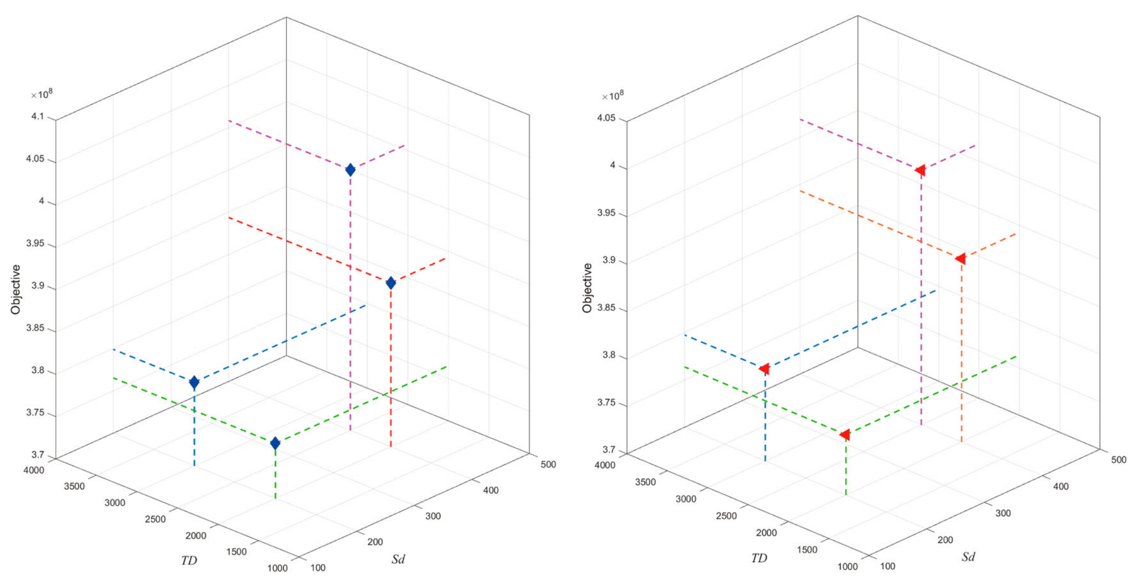
Based on the numerical experiments and result analyses, the following managerial insights for decision-makers in industry were obtained:

- The SS&OA schemes provided by the proposed two-stage DR Mean-CVaR model and its nominal model

Table 9. The optimal SS&OA schemes identified by the Polyhedral and Box models under $N = 24$.

• Polyhedral model with $\delta = 0.2, \alpha = 0.5, \varepsilon = 0.9$ Obj: 399,026,223.39												
Main supplier: H_2, q_2: $1.107K_1$												
$(q_{2,s})_{s\in[24]}$	$1.107K_1$	$1.107K_1$	$1.107K_1$	$9.762K_2$	$1.107K_1$	$1.107K_1$	$1.107K_1$	$1.107K_1$	$1.107K_1$	$1.107K_1$	$1.107K_1$	$1.107K_1$
	$1.107K_1$	$9.034K_2$	$9.325K_2$	$1.107K_1$	$9.034K_2$	$1.107K_1$	$1.107K_1$	$8.742K_2$	$9.908K_2$	$1.064K_1$	$1.107K_1$	$1.107K_1$
$(z_{2,s})_{s\in[24]}$	0	0	0	0	0	$2.398K_2$	0	0	0	0	89279	0
	0	0	0	$9.925K_2$	0	0	$6.914K_2$	0	0	0	0	0
Main supplier: H_4, q_4: $1.063K_1$												
$(q_{4,s})_{s\in[24]}$	$1.063K_1$	$1.063K_1$	$1.063K_1$	$1.063K_1$	$1.063K_1$	$1.039K_1$	$1.063K_1$	$1.063K_1$	$1.063K_1$	$1.063K_1$	$1.054K_1$	$1.063K_1$
	$1.063K_1$	$1.063K_1$	$9.484K_2$	$9.634K_2$	$1.063K_1$	$1.063K_1$	$9.935K_2$	$1.063K_1$	$1.063K_1$	$1.063K_1$	$1.063K_1$	$1.063K_1$
$(z_{4,s})_{s\in[24]}$	0	0	0	$1.311K_2$	0	0	0	0	0	0	0	0
	0	$2.04K_2$	0	0	$2.04K_2$	0	0	$2.331K_2$	0	$4.371K_3$	0	0
Backup supplier: H_1												
$(q'_{1,s})_{s\in[24]}$	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	$2.891K_2$	0	0	0	0	0	$1.166K_2$	0	0	0
• Box model with $\sigma = 0.3, \alpha = 0.5, \varepsilon = 0.9$ Obj: 395,943,902.05												
Main supplier: H_2, q_2: $9.808K_2$												
$(q_{2,s})_{s\in[24]}$	$9.808K_2$	$9.808K_2$	$9.808K_2$	$9.762K_2$	$9.808K_2$	$9.808K_2$	$9.808K_2$	$9.808K_2$	$9.808K_2$	$9.808K_2$	$9.808K_2$	$9.808K_2$
	$9.808K_2$	$9.034K_2$	$9.325K_2$	$9.808K_2$	$9.038K_2$	$9.808K_2$	$9.808K_2$	$8.742K_2$	$9.808K_2$	$9.808K_2$	$9.808K_2$	$9.808K_2$
$(z_{2,s})_{s\in[24]}$	0	0	0	0	0	$1.505K_2$	0	0	0	0	$1.355K_2$	0
	$7.527K_3$	0	0	$2.258K_2$	0	0	$1.957K_2$	0	0	0	0	$1.204K_2$
Main supplier: H_4, q_4: $1.189K_1$												
$(q_{4,s})_{s\in[24]}$	$1.189K_1$	$1.189K_1$	$1.189K_1$	$1.189K_1$	$1.189K_1$	$1.039K_1$	$1.189K_1$	$1.189K_1$	$1.189K_1$	$1.189K_1$	$1.054K_1$	$1.189K_1$
	$1.114K_1$	$1.189K_1$	$9.484K_2$	$9.634K_2$	$1.189K_1$	$1.189K_1$	$9.935K_2$	$1.189K_1$	$1.159K_1$	$1.189K_1$	$1.189K_1$	$1.069K_1$
$(z_{4,s})_{s\in[24]}$	0	0	0	45817	0	0	0	0	0	0	0	0
	0	$7.743K_3$	0	0	$7.743K_3$	0	0	$1.066K_2$	0	0	0	0
Backup supplier: H_1												
$(q'_{1,s})_{s\in[24]}$	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	$2.891K_2$	0	0	0	0	0	$3.011K_3$	0	0	0

Note: Obj denotes objective, K_1 denotes 10^7, K_2 denotes 10^6, K_3 denotes 10^5.



(a) (b)

Figure 9. The objectives of models with $\alpha = 0.5, \varepsilon = 0.9$ under different (*Sd, TD*). (a) Polyhedral model with $\delta = 0.2$ (b) Box model with $\sigma = 0.3$.

Table 10. Experimental results of the Polyhedral and Box models under different (Sd, TD).

	Supplier		Total	Obj	
(Sd, TD)	Main	Backup	distance	Polyhedral	Box
(200, 2000)	H_2, H_5	H_1	2239.3	$3.765K_4$	$3.763K_4$
(400, 2000)	H_1, H_2	H_4	2361.4	$3.893K_4$	$3.893K_4$
(400, 2500)	H_2, H_4	H_3	2590.5	$4.008K_4$	$3.968K_4$
(200, 3000)	H_2, H_5	H_1, H_4	3768.4	$3.798K_4$	$3.797K_4$

Note: Obj denotes objective, K_4 denotes 10^8.

are different. This implies that the uncertain probabilities have a significant impact on the optimal SS&OA scheme. Under the condition that probabilities of occurrence for disruption scenarios are ambiguous, the application of the SS&OA scheme provided by the nominal model may not obtain the ideal result. If the decision-maker cannot obtain the exact probabilities of occurrence for disruption scenarios and wants to immunise against the influence of the uncertain probabilities, he or she can apply the proposed two-stage DR Mean-CVaR model to make informed decisions. The robust optimal decision provided by our model is resistant to the uncertain probabilities at a small price. The proposed two-stage DR Mean-CVaR model is an important enhancement of the methods in the literature.

- The values of the trade-off parameter and confidence level parameter reflect the risk preference of the decision-maker and affect the optimal objective and SS&OA scheme of the two-stage DR Mean-CVaR model. A smaller trade-off parameter indicates that the decision-maker emphasises the average level at which the cost exceeds the given VaR value. A higher confidence level indicates that the decision-maker is risk averse. If the decision-maker pays more attention to risk, then he or she should choose a smaller trade-off parameter and a larger confidence level parameter. Otherwise, he or she should choose a larger trade-off parameter and a smaller confidence level parameter.
- By performing the sensitivity analysis of the sizes of ambiguity sets, it can be concluded that the sizes obviously affect the optimal SS&OA solution from the proposed two-stage DR Mean-CVaR model. Different scales of ambiguity sets produce different optimal values and prices to resist uncertainty. A larger ambiguity set includes a larger range of probability vector disturbances. The decision-maker should choose reasonable ambiguity sets according to the enterprise's risk tolerance. If he or she wants the optimal decision to immunise against the influence of the larger uncertainty, the larger scale of ambiguity sets should be chosen. This is a key point and is also a new finding.
- From the sensitivity analysis of the number of selected scenarios, it is a critical step to select scenarios when constructing the ambiguity sets in the proposed robust model. This number significantly affects the optimal SS&OA schemes provided by the Polyhedral and Box models. An increase in this number implies that the obtained SS&OA schemes provided by the Polyhedral and Box models are suitable for more scenarios. However, the cost also increases with respect to this number. Therefore, the decision-maker should make a trade-off between the number of selected scenarios and cost and rationally filter scenarios according to his or her risk preference. The conservative decision-maker should choose more scenarios. Otherwise, the decision-maker can choose fewer scenarios.
- Once a disruption occurs in a supply chain network, there may be a series of ripple effects (Ivanov 2022a; Ivanov 2022b; Rozhkov et al. 2022). As mentioned in the literature (Dolgui, Ivanov, and Rozhkov 2020a; Ivanov and Dolgui 2021a), it is essential to incorporate resilience strategies (e.g. cooperating with backup suppliers, surplus supply from the non-disrupted main suppliers, setting resilience distances) into the supply chain to avoid disruptions under multiple disruption risks. Taking the resilience distances as an example, if the decision-maker wants to make the robust optimal SS&OA scheme more resilient, then he or she should set a larger resilience distance.

7. Conclusions and future research

This paper studied the SS&OA problem under disruption risks. A series of disruption scenarios was used to describe disruption risks. Since it is usually unrealistic to assume that the probabilities of occurrence for disruption scenarios are deterministic in our lives, we addressed that these probabilities are uncertain in the current study. A novel two-stage DR Mean-CVaR SS&OA model for risk-averse decision-makers in industry was developed. For this purpose, we incorporated three resilience measures into the proposed model to increase the resilience of the supply chain. The main finding, that the probabilities' uncertainty indeed has a significant impact on the optimal SS&OA scheme, was obtained. For the SS&OA problem with uncertain probabilities of occurrence for disruption scenarios in other contexts (e.g. companies, industries, countries), the developed two-stage DR Mean-CVaR model is also valid.

The developed method adopts a flexible structure and has the following merits: (i) the established model can be applied to the case where the probabilities of occurrence for disruption scenarios are uncertain, and (ii) the risk-averse decision-maker in industry can apply the developed model to balance the average cost and the level of risk. As a consequence, this study has practical implications and fills gaps in the current literature. Moreover, since the SS&OA problem was based on uncertain probabilities of occurrence for disruption scenarios, it aims to balance the average cost and the level of risk, which was not addressed in the literature. From this point of view, this study also has theoretical implications.

The procedure for constructing the ambiguity set was developed and the Polyhedral and Box ambiguity sets were constructed to characterise the uncertain probabilities. The Lagrange and linear duality theories were adopted to transform the proposed two-stage DR Mean-CVaR model into two MILP models under the constructed ambiguity sets, which can be solved by general commercial software. Therefore, the proposed two-stage DR Mean-CVaR model is practical.

The manufacturer of Huawei cell phones located in Changsha was used as a case to conduct some numerical experiments and to illustrate the feasibility and effectiveness of the proposed two-stage DR Mean-CVaR model. The experimental results show that the proposed two-stage DR method is not only feasible, but can also provide a robust SS&OA solution to protect against the influence of the uncertainty of the probabilities. In addition, a series of sensitivity analyses were performed regarding some parameters in the proposed model, such as the trade-off coefficient and the confidence level parameter. According to the experimental results, some management insights are summarised for the decision-maker in production research.

The research had two limitations. The first is that the types of ambiguity set constructed were not diverse enough for characterising these uncertain probabilities of occurrence for disruption scenarios. The second is that only the uncertainty of the probability vector was dealt with. In practice, there may be uncertainties in other model parameters.

There are some suggestions in terms of future research. The first extension is to add more resilience strategies (Hosseini et al. 2019b) to the SS&OA model and then further analyse which strategy has the greatest impact on the resilience of the supply chain (Moosavi and Hosseini 2021) under the uncertain probabilities of occurrence for disruption scenarios. The second interesting extension is to construct other types of ambiguity sets to characterise these uncertain probabilities of occurrence for disruption scenarios. The third interesting extension is to investigate the SS&OA problem under other uncertain model parameters, such as demand and supply capacity. Meanwhile, when distributional information about these uncertain parameters is partially known, the decision-maker can use fuzzy DRO method (Liu, Chen, and Liu 2021) or stochastic DRO method (Delage and Ye 2010) to address the SS&OA problem.

Notes

1. https://finance.eastmoney.com/a/202106111957465278.html
2. https://zhuanlan.zhihu.com/p/157520487
3. https://www.idc.com/getdoc.jsp?containerId=prAP47424421
4. https://map.baidu.com/

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Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article, further reasonable inquiries can be directed to the corresponding author.

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Appendices

Appendix 1: Handling non-linear constraint

Proposition A.1: *Let $r^1_{ij} \in \{0,1\}, \forall i,j \in I$. Non-linear item $\sum_{i\in I}\sum_{j\in I, i<j} x_i x_j d_{ij}$ equals linear item $\sum_{i\in I}\sum_{j\in I,i<j} r^1_{ij} d_{ij}$, where r^1_{ij}, x_i, x_j satisfy the following linear constraints:*

$$r^1_{ij} \le x_i, \quad \forall i,j \in I, \tag{A1}$$

$$r^1_{ij} \le x_j, \quad \forall i,j \in I, \tag{A2}$$

$$r^1_{ij} \ge x_i + x_j - 1, \quad \forall i,j \in I. \tag{A3}$$

Proof: From constraints (A1)–(A3), for any given $i,j \in I, i<j$, it is known that the combination (x_i, x_j, r^1_{ij}) takes values in one of the following four cases: (i) if $x_i = 0, x_j = 0$, then $r^1_{ij} = 0$; (ii) if $x_i = 1, x_j = 0$, then $r^1_{ij} = 0$; (iii) if $x_i = 0, x_j = 1$, then $r^1_{ij} = 0$; (iv) if $x_i = 1, x_j = 1$, then $r^1_{ij} = 1$. In all four cases, $x_i x_j d_{ij} = r^1_{ij} d_{ij}$ holds. Hence, $\sum_{i\in I}\sum_{j\in I,i<j} x_i x_j d_{ij}$ equals $\sum_{i\in I}\sum_{j\in I,i<j} r^1_{ij} d_{ij}$. ■

The following Corollary 1 gives the re-representation of non-linear constraint (1).

Corollary A.1: *Non-linear constraint (1) is equivalent to the following linear constraints:*

$$TD \le \sum_{i\in I}\sum_{j\in I,i<j} r^1_{ij} d_{ij} + \sum_{i\in I}\sum_{j\in I} r^2_{ij} d_{ij} + \sum_{i\in I}\sum_{j\in I,i<j} r^3_{ij} d_{ij}, \tag{A4}$$

$$r^2_{ij} \le x_i, \quad \forall i,j \in I, \tag{A5}$$

$$r^2_{ij} \le x'_j, \quad \forall i,j \in I, \tag{A6}$$

$$r^2_{ij} \ge x_i + x'_j - 1, \quad \forall i,j \in I, \tag{A7}$$

$$r^3_{ij} \le x'_i, \quad \forall i,j \in I, \tag{A8}$$

$$r^3_{ij} \le x'_j, \quad \forall i,j \in I, \tag{A9}$$

$$r^3_{ij} \ge x'_i + x'_j - 1, \quad \forall i,j \in I, \tag{A10}$$

$$r^1_{ij}, r^2_{ij}, r^3_{ij} \in \{0,1\}, \quad \forall i,j \in I, \tag{A11}$$

$$\text{Constraints(A1)-(A3).}$$

Proof: According to the proof of Proposition 1, it is known that $x_i x'_j d_{ij} = r^2_{ij} d_{ij}$ and $x'_i x'_j d_{ij} = r^3_{ij} d_{ij}$, which complete the proof of the corollary.

For convenience, let $\boldsymbol{r} = (\boldsymbol{r}^1, \boldsymbol{r}^2, \boldsymbol{r}^3)$, $\boldsymbol{r}^1 = (r^1_{ij})_{i,j\in I}$, $\boldsymbol{r}^2 = (r^2_{ij})_{i,j\in I}$, and $\boldsymbol{r}^3 = (r^3_{ij})_{i,j\in I}$. As a result, the non-linear constraint (1) can be linearised. ■

Appendix 2: The proof of Theorem 1

Proof: We first deal with the first item $\max_{\mathbf{P}\in\mathcal{P}} \alpha\, \mathrm{TC}_2^{\mathrm{T}}\, \mathbf{P}$ in (25). The following relationship can be obtained:

$$\alpha \max_{\mathbf{P}\in\mathcal{P}} \mathrm{TC}_2^{\mathrm{T}}\, \mathbf{P} \Leftrightarrow \alpha\, \mathrm{TC}_2^{\mathrm{T}}\, \mathbf{P}_0 + \alpha \max_{\boldsymbol{\xi}} \{\mathrm{TC}_2^{\mathrm{T}}\, \mathbf{A}\boldsymbol{\xi} | \boldsymbol{e}^{\mathrm{T}}\mathbf{A}\boldsymbol{\xi} = 0, \mathbf{P}_0 + \mathbf{A}\boldsymbol{\xi} \ge 0, ||\boldsymbol{\xi}||_1 \le 1\}.$$

We next handle the above inner maximisation problem. It is known that the following maximisation problem:

$$\max_{\boldsymbol{\xi}} \{\mathrm{TC}_2^{\mathrm{T}}\, \mathbf{A}\boldsymbol{\xi} | \boldsymbol{e}^{\mathrm{T}}\mathbf{A}\boldsymbol{\xi} = 0, \mathbf{P}_0 + \mathbf{A}\boldsymbol{\xi} \ge 0, ||\boldsymbol{\xi}||_1 \le 1\}$$

is equivalent to the minimisation problem

$$-\min_{\boldsymbol{\xi}} \{-\mathrm{TC}_2^{\mathrm{T}}\, \mathbf{A}\boldsymbol{\xi} | \boldsymbol{e}^{\mathrm{T}}\mathbf{A}\boldsymbol{\xi} = 0, \mathbf{P}_0 + \mathbf{A}\boldsymbol{\xi} \ge 0, ||\boldsymbol{\xi}||_1 \le 1\}.$$

The Lagrange function of the above minimisation optimisation problem is as follows:

$$\mathcal{L}(\theta, \boldsymbol{\Psi}, \mu, \boldsymbol{\xi}) = -\mathrm{TC}_2^{\mathrm{T}}\, \mathbf{A}\boldsymbol{\xi} + \boldsymbol{\Psi}^{\mathrm{T}}(-\mathbf{P}_0 - \mathbf{A}\boldsymbol{\xi}) + \mu\boldsymbol{e}^{\mathrm{T}}\mathbf{A}\boldsymbol{\xi} + \theta(||\boldsymbol{\xi}||_1 - 1),$$

where $(\theta, \boldsymbol{\Psi}, \mu) \in \mathbb{R} \times \mathbb{R}^{|S|} \times \mathbb{R}$.

Then the Lagrange dual function is given by

$$\begin{aligned} g(\theta, \boldsymbol{\Psi}, \mu) &= \min_{\boldsymbol{\xi}} \mathcal{L}(\theta, \boldsymbol{\Psi}, \mu, \boldsymbol{\xi}) \\ &= (-\mathbf{P}_0^{\mathrm{T}}\boldsymbol{\Psi} - \theta) \\ &\quad - \max_{\boldsymbol{\xi}} [(\mathbf{A}^{\mathrm{T}}\, \mathrm{TC}_2 + \mathbf{A}^{\mathrm{T}}\boldsymbol{\Psi} - \mathbf{A}^{\mathrm{T}}\boldsymbol{e}\mu)^{\mathrm{T}}\boldsymbol{\xi} - \theta||\boldsymbol{\xi}||_1] \\ &= (-\mathbf{P}_0^{\mathrm{T}}\boldsymbol{\Psi} - \theta) - f^*(\mathbf{A}^{\mathrm{T}}\, \mathrm{TC}_2 + \mathbf{A}^{\mathrm{T}}\boldsymbol{\Psi} - \mathbf{A}^{\mathrm{T}}\boldsymbol{e}\mu), \end{aligned}$$

where

$$f^*(\mathbf{A}^{\mathrm{T}}\, \mathrm{TC}_2 + \mathbf{A}^{\mathrm{T}}\boldsymbol{\Psi} - \mathbf{A}^{\mathrm{T}}\boldsymbol{e}\mu) = \begin{cases} 0, & ||\mathbf{A}^{\mathrm{T}}\, \mathrm{TC}_2 + \mathbf{A}^{\mathrm{T}}\boldsymbol{\Psi} - \mathbf{A}^{\mathrm{T}}\boldsymbol{e}\mu||_\infty \le \theta, \\ \infty, & \text{otherwise,} \end{cases}$$

is the conjugate function of $f^*(\boldsymbol{\xi}) = \theta||\boldsymbol{\xi}||$ (see Boyd, Boyd, and Vandenberghe (2004)). Since the Lagrange dual function yields lower bounds for any $\boldsymbol{\Psi} \ge 0$ and $\theta \ge 0$, we obtain that $\max_{\theta,\boldsymbol{\Psi},\mu} g(\theta, \boldsymbol{\Psi}, \mu)$ is equivalent to the following maximisation problem:

$$\begin{aligned} \max_{\theta,\boldsymbol{\Psi},\mu} \quad & -\mathbf{P}_0^{\mathrm{T}}\boldsymbol{\Psi} - \theta \\ \text{s.t.} \quad & ||\mathbf{A}^{\mathrm{T}}\, \mathrm{TC}_2 + \mathbf{A}^{\mathrm{T}}\boldsymbol{\Psi} - \mathbf{A}^{\mathrm{T}}\boldsymbol{e}\mu||_\infty \le \theta, \\ & \boldsymbol{\Psi} \ge 0, \theta \ge 0. \end{aligned}$$

Further, the following equivalent relationships are established:

$$\begin{aligned} & \max_{\mathbf{P}\in\mathcal{P}} \alpha\, \mathrm{TC}_2^{\mathrm{T}}\, \mathbf{P} \\ & \Leftrightarrow \alpha\, \mathrm{TC}_2^{\mathrm{T}}\, \mathbf{P}_0 \\ & \quad - \alpha \begin{cases} \max_{\theta,\boldsymbol{\Psi},\mu} & -\mathbf{P}_0^{\mathrm{T}}\boldsymbol{\Psi} - \theta \\ \text{s.t.} & ||\mathbf{A}^{\mathrm{T}}\, \mathrm{TC}_2 + \mathbf{A}^{\mathrm{T}}\boldsymbol{\Psi} - \mathbf{A}^{\mathrm{T}}\boldsymbol{e}\mu||_\infty \le \theta, \\ & \boldsymbol{\Psi} \ge 0, \theta \ge 0. \end{cases} \\ & \Leftrightarrow \begin{cases} \min_{\theta,\boldsymbol{\Psi},\mu} & \alpha(\mathrm{TC}_2^{\mathrm{T}}\, \mathbf{P}_0 + \mathbf{P}_0^{\mathrm{T}}\Psi + \theta) \\ \text{s.t.} & ||\mathbf{A}^{\mathrm{T}}\, \mathrm{TC}_2 + \mathbf{A}^{\mathrm{T}}\boldsymbol{\Psi} - \mathbf{A}^{\mathrm{T}}\boldsymbol{e}\mu||_\infty \le \theta, \\ & \boldsymbol{\Psi} \ge 0, \theta \ge 0. \end{cases} \end{aligned}$$

Using the similar method, we get the following equivalent representation:

$$\begin{aligned}
&\max_{\mathbf{P}\in\mathcal{P}}(1-\alpha)\left\{\phi+\frac{1}{1-\varepsilon}\mathbf{t}^{\mathrm{T}}\mathbf{P}\right\}\\
&\Leftrightarrow (1-\alpha)\phi+\frac{1-\alpha}{1-\varepsilon}\mathbf{t}^{\mathrm{T}}\mathbf{P}_0\\
&\quad -\frac{1-\alpha}{1-\varepsilon}\begin{cases}\max\limits_{\hat{\theta},\hat{\boldsymbol{\Psi}},\hat{\mu}} & -\mathbf{P}_0^{\mathrm{T}}\hat{\boldsymbol{\Psi}}-\hat{\theta}\\ \text{s.t.} & ||\mathbf{A}^{\mathrm{T}}\mathbf{t}+\mathbf{A}^{\mathrm{T}}\hat{\boldsymbol{\Psi}}-\mathbf{A}^{\mathrm{T}}\boldsymbol{e}\hat{\mu}||_\infty\le\hat{\theta},\\ & \hat{\boldsymbol{\Psi}}\ge 0,\hat{\theta}\ge 0\end{cases}\\
&\Leftrightarrow\begin{cases}\min\limits_{\hat{\theta},\hat{\boldsymbol{\Psi}},\hat{\mu}} & (1-\alpha)\left\{\phi+\dfrac{\mathbf{t}^{\mathrm{T}}\mathbf{P}_0+\mathbf{P}_0^{\mathrm{T}}\hat{\Psi}+\hat{\theta}}{1-\varepsilon}\right\}\\ \text{s.t.} & ||\mathbf{A}^{\mathrm{T}}\mathbf{t}+\mathbf{A}^{\mathrm{T}}\hat{\boldsymbol{\Psi}}-\mathbf{A}^{\mathrm{T}}\mathbf{e}\hat{\mu}||_\infty\le\hat{\theta},\\ & \hat{\boldsymbol{\Psi}}\ge 0,\hat{\theta}\ge 0.\end{cases}
\end{aligned}$$

The proof of Theorem 1 is complete. ■

Appendix 3: the proof of Theorem 2

Proof: We first need to deal with the following equivalent reformulation:

$$\begin{aligned}
&\max_{\mathbf{P}\in\mathcal{P}}\alpha\,\mathrm{TC}_2^{\mathrm{T}}\,\mathbf{P}\Leftrightarrow\alpha\,\mathrm{TC}_2^{\mathrm{T}}\,\mathbf{P}_0\\
&\quad+\alpha\max_{\boldsymbol{\xi}}\{\mathrm{TC}_2^{\mathrm{T}}\,\boldsymbol{\xi}|\mathbf{e}^{\mathrm{T}}\boldsymbol{\xi}=0,\boldsymbol{\xi}_L\le\boldsymbol{\xi}\le\boldsymbol{\xi}_U\}.
\end{aligned}$$

For $\max\limits_{\boldsymbol{\xi}}\{\mathrm{TC}_2^{\mathrm{T}}\,\boldsymbol{\xi}|\mathbf{e}^{\mathrm{T}}\boldsymbol{\xi}=0,\boldsymbol{\xi}_L\le\boldsymbol{\xi}\le\boldsymbol{\xi}_U\}$, according to linear programming duality theory, its dual programming problem is

$$\begin{aligned}
\min_{\tau,\boldsymbol{\gamma},\boldsymbol{\pi}}\quad & \boldsymbol{\xi}_U^{\mathrm{T}}\boldsymbol{\pi}-\boldsymbol{\xi}_L^{\mathrm{T}}\boldsymbol{\gamma}\\
\text{s.t.}\quad & \boldsymbol{e}\tau+\boldsymbol{\pi}-\boldsymbol{\gamma}=\mathrm{TC}_2,\\
& \boldsymbol{\pi}\ge 0,\\
& \boldsymbol{\gamma}\ge 0,
\end{aligned}$$

where $(\tau,\boldsymbol{\pi},\boldsymbol{\gamma})\in\mathbb{R}\times\mathbb{R}^{|S|}\times\mathbb{R}^{|S|}$ is the dual variables.

Finally, we get the following equivalent reformulation:

$$\max_{\mathbf{P}\in\mathcal{P}}\alpha\,\mathrm{TC}_2^{\mathrm{T}}\,\mathbf{P}\Leftrightarrow\begin{cases}\min\limits_{\tau,\boldsymbol{\gamma},\boldsymbol{\pi}} & \alpha(\mathrm{TC}_2^{\mathrm{T}}\,\mathbf{P}_0+\boldsymbol{\xi}_U^{\mathrm{T}}\boldsymbol{\pi}-\boldsymbol{\xi}_L^{\mathrm{T}}\boldsymbol{\gamma})\\ \text{s.t.} & \boldsymbol{e}\tau+\boldsymbol{\pi}-\boldsymbol{\gamma}=\mathrm{TC}_2,\\ & \boldsymbol{\pi}\ge 0,\\ & \boldsymbol{\gamma}\ge 0.\end{cases}$$

Using the similar method, we get the next equivalent reformulation:

$$\begin{aligned}
&\max_{\mathbf{P}\in\mathcal{P}}(1-\alpha)\left\{\phi+\frac{\mathbf{t}^{\mathrm{T}}\mathbf{P}}{1-\varepsilon}\right\}\\
&\Leftrightarrow\begin{cases}\min\limits_{\hat{\tau},\hat{\boldsymbol{\gamma}},\hat{\boldsymbol{\pi}}} & (1-\alpha)\left\{\phi+\dfrac{\mathbf{t}^{\mathrm{T}}\mathbf{P}_0+\boldsymbol{\xi}_U^{\mathrm{T}}\hat{\boldsymbol{\pi}}-\boldsymbol{\xi}_L^{\mathrm{T}}\hat{\boldsymbol{\gamma}}}{1-\varepsilon}\right\}\\ \text{s.t.} & \boldsymbol{e}\hat{\tau}+\hat{\boldsymbol{\pi}}-\hat{\boldsymbol{\gamma}}=\mathbf{t},\\ & \hat{\boldsymbol{\pi}}\ge 0,\\ & \hat{\boldsymbol{\gamma}}\ge 0.\end{cases}
\end{aligned}$$

The proof of Theorem 2 is complete. ■